Bipartite quantum measurements with optimal single-sided distinguishability

Jakub Czartowski[1] Karol Życzkowski[1]

[1]Jagiellonian University
Faculty of Physics, Astronomy and Applied Computer Science
Institute for Theoretical Physics

jakub.czartowski@doctoral.uj.edu.pl

KCIK Symposium, Gdańsk/Kraków,
May 21, 2021

Overview

What are we looking for?

- The basic task in the work is to find bases with optimal distinguishability between any pair of states.

- The basic task in the work is to find bases with optimal distinguishability between any pair of states.
- Simple enough, right?

What are we looking for?

- The basic task in the work is to find bases with optimal distinguishability between any pair of states.
- Simple enough, right?
- However, once we add several additional restrictions...

What are we looking for?

- The basic task in the work is to find bipartite bases of dimension N^2 with optimal single-sided distinguishability between any pair of states.
- Simple enough, right?
- However, once we add several additional restrictions **the problem becomes not really simple at all!**

Helstrom’s theorem

- The premise of distinguishability is based on the Helstrom theorem for distinguishability

Helstrom's theorem

- The premise of distinguishability is based on the Helstrom theorem for distinguishability

Theorem (Helstrom's theorem)

Given two arbitrary quantum states $\rho, \sigma \in \Omega_N$, the probability of distinguishing between the two in a single-shot experiment is upperbound by

$$p \leq \frac{1}{2}\left(1 + \frac{1}{2} D_{tr}(\rho, \sigma)\right), \tag{1}$$

which is saturated by measurement based on eigenvectors of the difference of the quantum states, $\rho - \sigma$ given by measurement operators

$$P_\rho = \sum_{\lambda_i > 0} |\lambda_i\rangle\langle\lambda_i|, \qquad P_\sigma = \sum_{\lambda_1 < 0} |\lambda_i\rangle\langle\lambda_i|, \qquad P_0 = \mathbb{I} - P_\rho - P_\sigma. \tag{2}$$

Helstrom's theorem

- Applied to two states from an orthonormal basis, the result is trivial, as $p \leq 1$ and the states can be distinguished with certainty.

Helstrom's theorem

- Applied to two states from an orthonormal basis, the result is trivial, as $p \leq 1$ and the states can be distinguished with certainty.
- The same statement for the reductions to Alice's subspace $\rho_{Ai} = \mathrm{Tr}_B \left|i\right\rangle\!\left\langle i\right|$ and likewise for Bob's, turns out to be nontrivial.

Helstrom's theorem

- Applied to two states from an orthonormal basis, the result is trivial, as $p \leq 1$ and the states can be distinguished with certainty.
- The same statement for the reductions to Alice's subspace $\rho_{Ai} = \mathrm{Tr}_B |i\rangle\langle i|$ and likewise for Bob's, turns out to be nontrivial.
- Take the standard computational basis on dimension N^2, $|ij\rangle = |i\rangle \otimes |j\rangle$. It yields only N distinguishable points in reductions!

Helstrom's theorem

- Applied to two states from an orthonormal basis, the result is trivial, as $p \leq 1$ and the states can be distinguished with certainty.
- The same statement for the reductions to Alice's subspace $\rho_{A\,i} = \mathrm{Tr}_B\,|i\rangle\langle i|$ and likewise for Bob's, turns out to be nontrivial.
- Take the standard computational basis on dimension N^2, $|i\,j\rangle = |i\rangle \otimes |j\rangle$. It yields only N distinguishable points in reductions!
- With setting a restraint that any two states should be equally distinguishable in reductions,

$$D_{tr}(\rho_{A\,i}, \rho_{A\,j}) = D(1 - \delta_{ij}) \tag{3}$$

for some constant D we see, that we are actually looking for maximal regular simplices.

Helstrom's theorem

- Applied to two states from an orthonormal basis, the result is trivial, as $p \leq 1$ and the states can be distinguished with certainty.
- The same statement for the reductions to Alice's subspace $\rho_{A\,i} = \mathrm{Tr}_B\, |i\rangle\langle i|$ and likewise for Bob's, turns out to be nontrivial.
- Take the standard computational basis on dimension N^2, $|i\,j\rangle = |i\rangle \otimes |j\rangle$. It yields only N distinguishable points in reductions!
- With setting a restraint that any two states should be equally distinguishable in reductions,

$$D_{tr}(\rho_{A\,i}, \rho_{A\,j}) = D(1 - \delta_{ij}) \tag{3}$$

  for some constant D we see, that we are actually looking for maximal regular simplices.
- This seems to be reminiscent of...

- ...reminiscent of SIC-POVMs, one of the most prominent simplicial structures in quantum information.

- ...reminiscent of SIC-POVMs, one of the most prominent simplicial structures in quantum information.
- A SIC in dimension N is defined by a set of N^2 states $\{|\psi_i\rangle\}_{i=1}^{N^2}$ such that their overlaps are constant,

$$|\langle i_0|j_0\rangle|^2 = \frac{1+N\delta_{ij}}{1+N}. \tag{4}$$

- ...reminiscent of SIC-POVMs, one of the most prominent simplicial structures in quantum information.
- A SIC in dimension N is defined by a set of N^2 states $\{|\psi_i\rangle\}_{i=1}^{N^2}$ such that their overlaps are constant,

$$|\langle i_0|j_0\rangle|^2 = \frac{1+N\delta_{ij}}{1+N}. \tag{4}$$

- A simplest example of SIC-POVM is the one for a qubit,

$$|1_0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad |2_0\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\\sqrt{2}\end{pmatrix} \quad |3_0\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\\sqrt{2}e^{i\frac{2\pi}{3}}\end{pmatrix} \quad |4_0\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\\sqrt{2}e^{-i\frac{2\pi}{3}}\end{pmatrix} \tag{5}$$

The simplest example – Gisin's EJM

- First example of our optimal bases is rather simple to write down, using $\lambda = \frac{1}{4}\left(2+\sqrt{3}\right)$ and orthogonal complements, $\langle i_0 | i_1 \rangle = 0$,

The simplest example – Gisin's EJM

- First example of our optimal bases is rather simple to write down, using $\lambda = \frac{1}{4}\left(2+\sqrt{3}\right)$ and orthogonal complements, $\langle i_0 | i_1 \rangle = 0$,

$$
\begin{aligned}
|\psi_1\rangle &= \sqrt{\lambda}\,|1_0\rangle \otimes |1_0\rangle - \sqrt{1-\lambda}\,|1_1\rangle \otimes |1_1\rangle \\
|\psi_2\rangle &= \sqrt{\lambda}\,|2_0\rangle \otimes |2_0\rangle - \sqrt{1-\lambda}\,|2_1\rangle \otimes |2_1\rangle \\
|\psi_3\rangle &= \sqrt{\lambda}\,|3_0\rangle \otimes |4_0\rangle - \sqrt{1-\lambda}\,|3_1\rangle \otimes |4_1\rangle \\
|\psi_4\rangle &= \sqrt{\lambda}\,|4_0\rangle \otimes |3_0\rangle - \sqrt{1-\lambda}\,|4_1\rangle \otimes |3_1\rangle \,.
\end{aligned}
\tag{6}
$$

The simplest example – Gisin's EJM

- First example of our optimal bases is rather simple to write down, using $\lambda = \frac{1}{4}\left(2+\sqrt{3}\right)$ and orthogonal complements, $\langle i_0 | i_1 \rangle = 0$,

$$
\begin{aligned}
|\psi_1\rangle &= \sqrt{\lambda}\,|1_0\rangle \otimes |1_0^*\rangle - \sqrt{1-\lambda}\,|1_1\rangle \otimes |1_1^*\rangle \\
|\psi_2\rangle &= \sqrt{\lambda}\,|2_0\rangle \otimes |2_0^*\rangle - \sqrt{1-\lambda}\,|2_1\rangle \otimes |2_1^*\rangle \\
|\psi_3\rangle &= \sqrt{\lambda}\,|3_0\rangle \otimes |3_0^*\rangle - \sqrt{1-\lambda}\,|3_1\rangle \otimes |3_1^*\rangle \\
|\psi_4\rangle &= \sqrt{\lambda}\,|4_0\rangle \otimes |4_0^*\rangle - \sqrt{1-\lambda}\,|4_1\rangle \otimes |4_1^*\rangle
\end{aligned}
\tag{6}
$$

using $\left|i_j^*\right\rangle$ as component-wise complex conjugate.

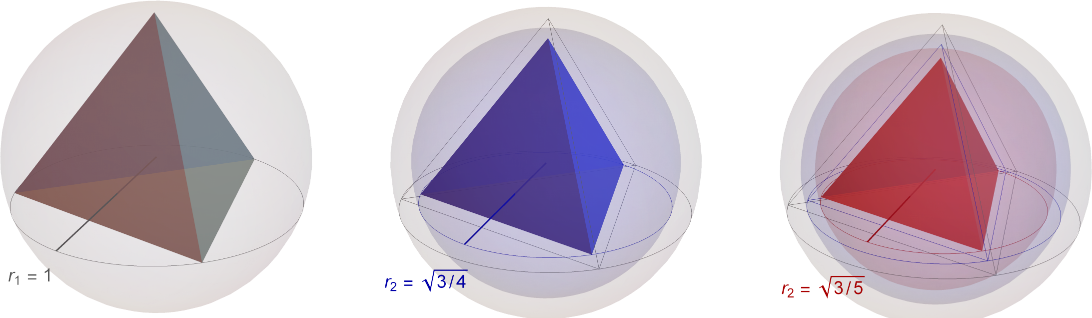
The simplest example – Gisin's EJM

- First example of our optimal bases is rather simple to write down, using $\lambda = \frac{1}{4}\left(2+\sqrt{3}\right)$ and orthogonal complements, $\langle i_0 | i_1 \rangle = 0$,

$$|\psi_i\rangle = \sqrt{\lambda}\,|i_0\rangle \otimes |i_0^*\rangle - \sqrt{1-\lambda}\,|i_1\rangle \otimes |i_1^*\rangle \tag{6}$$

using $\left|i_j^*\right\rangle$ as component-wise complex conjugate.

- It turns out that this solution is exactly the same as Elegant Joint Measurement introduced earlier by Gisin in the context of quantum networks and bilocality.



One more way EJM is optimal

- Stumbling upon EJM, we asked: in what other way is it optimal?

One more way EJM is optimal

- Stumbling upon EJM, we asked: in what other way is it optimal?
- One can interpret it as a unitary operation U_4 and investigate its entangling power e_p and gate typicality g_t, defined in terms of gate entropy E(U)

$$e_p(U) = \frac{E(U) + E(US) - E(S)}{E(S)} \qquad g_t(U) = \frac{E(U) - E(US) + E(S)}{2E(S)} \tag{7}$$

with S being the SWAP gate and the gate entropy $E(U) = 1 - \frac{1}{N^4}\sum_{i=1}^{N^2} \lambda_i^2(U^R)$ is expressed in terms of the reshuffled matrix.

One more way EJM is optimal

- Stumbling upon EJM, we asked: in what other way is it optimal?
- One can interpret it as a unitary operation U_4 and investigate its entangling power e_p and gate typicality g_t, defined in terms of gate entropy E(U)

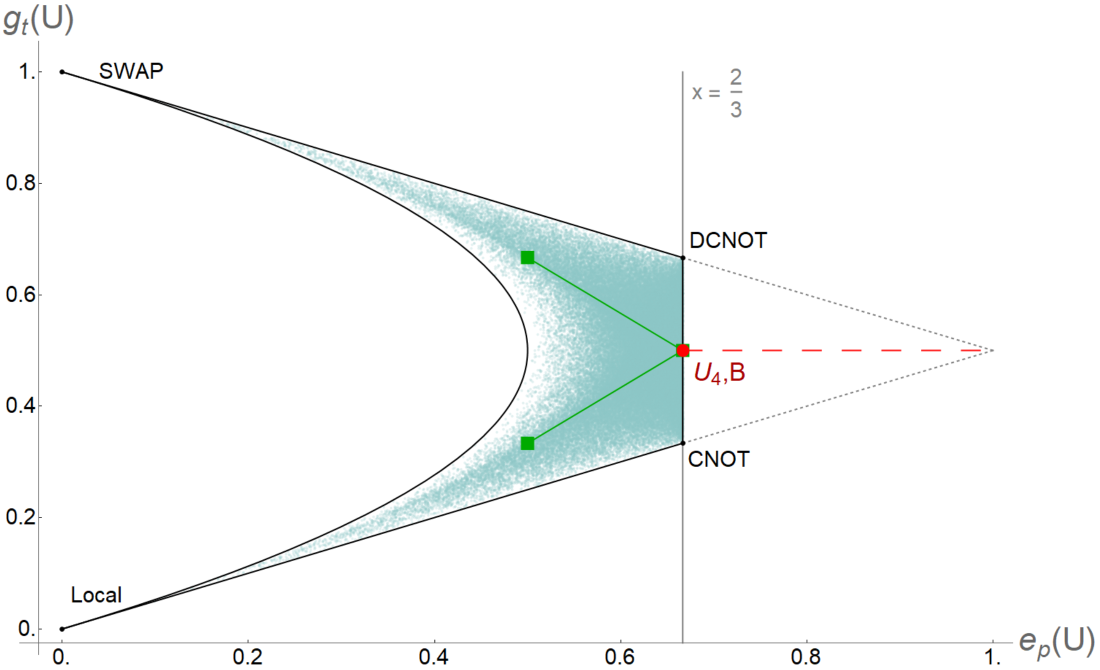
$$e_p(U) = \frac{E(U) + E(US) - E(S)}{E(S)} \qquad g_t(U) = \frac{E(U) - E(US) + E(S)}{2E(S)} \tag{7}$$

with S being the SWAP gate and the gate entropy $E(U) = 1 - \frac{1}{N^4}\sum_{i=1}^{N^2} \lambda_i^2(U^R)$ is expressed in terms of the reshuffled matrix.

- Turns out U_4 is related to a B gate,

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \tag{8}$$

shown to be optimal in terms of implementing nonlocal gates, which is closest to a bi-unitary gate, which does not exist for two qubits.


$g_t(U)$
1.
0.8
0.6
0.4
0.2
0.
SWAP
$x = \frac{2}{3}$
DCNOT
U_4,B
CNOT
Local
0.
0.2
0.4
0.6
0.8
1.
$e_p(U)$

The "almost" general solution

- We found that whenever there exists a SIC-POVM $\{|i_0\rangle\}_{i=1}^{N^2}$, one can construct a corresponding bipartite basis

$$|\psi_i\rangle = \sqrt{\lambda}\,|i_0\rangle\,|i_0^*\rangle - \sqrt{\frac{1-\lambda}{N-1}}\sum_{j=1}^{N-1}|i_j\rangle\left|i_j^*\right\rangle. \tag{9}$$

where $\langle i_a|i_b\rangle = \delta_{ab}$

The "almost" general solution

- We found that whenever there exists a SIC-POVM $\{|i_0\rangle\}_{i=1}^{N^2}$, one can construct a corresponding bipartite basis

$$|\psi_i\rangle = \sqrt{\lambda}\,|i_0\rangle\,|i_0^*\rangle - \sqrt{\frac{1-\lambda}{N-1}}\sum_{j=1}^{N-1}|i_j\rangle\left|i_j^*\right\rangle. \tag{9}$$

  where $\langle i_a|i_b\rangle = \delta_{ab}$
- The maximal Schmidt coefficient, directly related to maximal trace distances, is given by

$$\lambda = \frac{N^3 - N^2 - N + 2(N-1)\sqrt{N+1} + 2}{N^3} \tag{10}$$

The “almost” general solution

- We found that whenever there exists a SIC-POVM $\{|i_0\rangle\}_{i=1}^{N^2}$, one can construct a corresponding bipartite basis

$$|\psi_i\rangle = \sqrt{\lambda}\,|i_0\rangle\,|i_0^*\rangle - \sqrt{\frac{1-\lambda}{N-1}}\sum_{j=1}^{N-1}|i_j\rangle\left|i_j^*\right\rangle. \tag{9}$$

  where $\langle i_a|i_b\rangle = \delta_{ab}$
- The maximal Schmidt coefficient, directly related to maximal trace distances, is given by

$$\lambda = \frac{N^3 - N^2 - N + 2(N-1)\sqrt{N+1} + 2}{N^3} \tag{10}$$

- Such bases are optimal among the analyzed class of ”locally-simplex” bases and are conjectured to be optimal among all possible bases.

The "almost" general solution

- We found that whenever there exists a SIC-POVM $\{|i_0\rangle\}_{i=1}^{N^2}$, one can construct a corresponding bipartite basis

$$|\psi_i\rangle = \sqrt{\lambda}\,|i_0\rangle\,|i_0^*\rangle - \sqrt{\frac{1-\lambda}{N-1}}\sum_{j=1}^{N-1}|i_j\rangle\left|i_j^*\right\rangle. \tag{9}$$

  where $\langle i_a|i_b\rangle = \delta_{ab}$
- The maximal Schmidt coefficient, directly related to maximal trace distances, is given by

$$\lambda = \frac{N^3 - N^2 - N + 2(N-1)\sqrt{N+1} + 2}{N^3} \tag{10}$$

- Such bases are optimal among the analyzed class of "locally-simplex" bases and are conjectured to be optimal among all possible bases.
- Also an important takeaway is that

$$\lim_{N\to\infty}\lambda = 1 \tag{11}$$

  so in the limit of large dimension such a basis is approaching a tensor product of two independent local SIC-POVMs.

Example for qutrits

- For qutrits, the maximal Schmidt coefficient is $\lambda = \frac{25}{27}$.

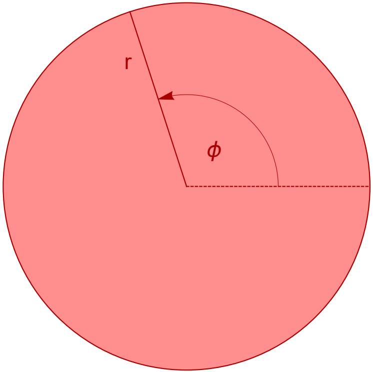
Example for qutrits

- For qutrits, the maximal Schmidt coefficient is $\lambda = \frac{25}{27}$.
- We start from a standard form of SIC-POVM for qutrits...

Example for qutrits

- For qutrits, the maximal Schmidt coefficient is $\lambda = \frac{25}{27}$.
- We start from a standard form of SIC-POVM for qutrits...
- But first, let us start from a clock notation showcase

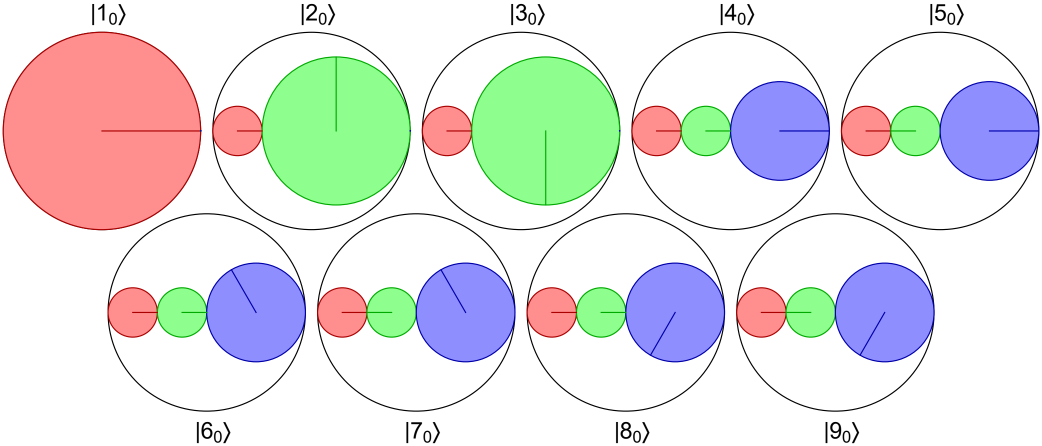
$\sqrt{r}\, e^{i\,\phi}$



so that $\sum r = 1$

Example for qutrits

- For qutrits, the maximal Schmidt coefficient is $\lambda = \frac{25}{27}$.
- We start from a standard form of SIC-POVM for qutrits...



Example for qutrits

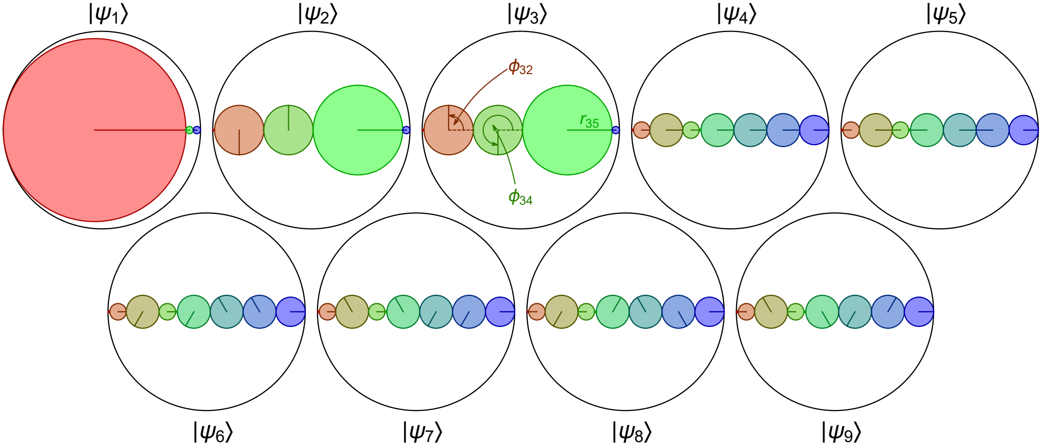
- For qutrits, the maximal Schmidt coefficient is $\lambda = \frac{25}{27}$.
- We start from a standard form of SIC-POVM for qutrits...
- ...and arrive at the desired basis for qutrits

$$U_9 = \begin{pmatrix} \langle\psi_1| \\ \langle\psi_2| \\ \langle\psi_3| \\ \langle\psi_4| \\ \langle\psi_5| \\ \langle\psi_6| \\ \langle\psi_7| \\ \langle\psi_8| \\ \langle\psi_9| \end{pmatrix} = \frac{1}{6\sqrt{6}} \begin{pmatrix} 10\sqrt{2} & 0 & 0 & 0 & -2\sqrt{2} & 0 & 0 & 0 & -2\sqrt{2} \\ \sqrt{2} & -3i\sqrt{6} & 0 & 3i\sqrt{6} & 7\sqrt{2} & 0 & 0 & 0 & -2\sqrt{2} \\ \sqrt{2} & 3i\sqrt{6} & 0 & -3i\sqrt{6} & 7\sqrt{2} & 0 & 0 & 0 & -2\sqrt{2} \\ \sqrt{2} & 3\sqrt{2} & 6 & 3\sqrt{2} & \sqrt{2} & 6 & 6 & 6 & 4\sqrt{2} \\ \sqrt{2} & -3\sqrt{2} & 6 & -3\sqrt{2} & \sqrt{2} & -6 & 6 & -6 & 4\sqrt{2} \\ \sqrt{2} & 3\sqrt{2} & 6\omega & 3\sqrt{2} & \sqrt{2} & 6\omega & 6\omega^2 & 6\omega^2 & 4\sqrt{2} \\ \sqrt{2} & 3\sqrt{2} & 6\omega^2 & 3\sqrt{2} & \sqrt{2} & 6\omega^2 & 6\omega & 6\omega & 4\sqrt{2} \\ \sqrt{2} & -3\sqrt{2} & 6\omega & -3\sqrt{2} & \sqrt{2} & -6\omega & 6\omega^2 & -6\omega^2 & 4\sqrt{2} \\ \sqrt{2} & -3\sqrt{2} & 6\omega^2 & -3\sqrt{2} & \sqrt{2} & -6\omega^2 & 6\omega & -6\omega & 4\sqrt{2} \end{pmatrix} \quad (12)$$

with $\omega = \exp(i2\pi/3)$.

Example for qutrits

- For qutrits, the maximal Schmidt coefficient is $\lambda = \frac{25}{27}$.
- We start from a standard form of SIC-POVM for qutrits...
- ...and arrive at the desired basis for qutrits



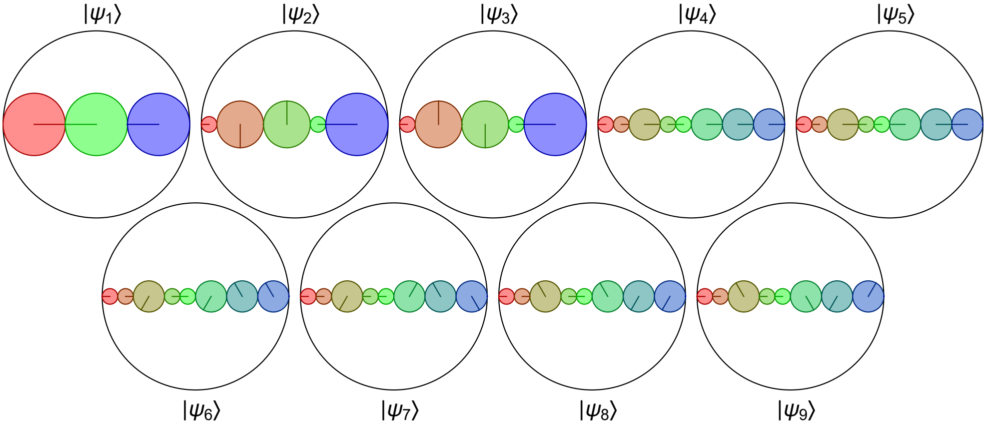
Maximally entangled counterpart

- For qutrits, it is also possible to construct a maximally entangled basis of a similar form, given by

$$U_9' = \frac{1}{2\sqrt{3}} \begin{pmatrix} 2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & -2 \\ -1 & -i\sqrt{3} & 0 & i\sqrt{3} & 1 & 0 & 0 & 0 & -2 \\ -1 & i\sqrt{3} & 0 & -i\sqrt{3} & 1 & 0 & 0 & 0 & -2 \\ -1 & 1 & \sqrt{2} & 1 & -1 & \sqrt{2} & \sqrt{2} & \sqrt{2} & 0 \\ -1 & -1 & \sqrt{2} & -1 & -1 & -\sqrt{2} & \sqrt{2} & -\sqrt{2} & 0 \\ -1 & 1 & \omega^2\sqrt{2} & 1 & -1 & \omega^2\sqrt{2} & \omega\sqrt{2} & \omega\sqrt{2} & 0 \\ -1 & -1 & \omega^2\sqrt{2} & -1 & -1 & -\omega^2\sqrt{2} & \omega\sqrt{2} & -\omega\sqrt{2} & 0 \\ -1 & 1 & \omega\sqrt{2} & 1 & -1 & \omega\sqrt{2} & \omega^2\sqrt{2} & \omega^2\sqrt{2} & 0 \\ -1 & -1 & \omega\sqrt{2} & -1 & -1 & -\omega\sqrt{2} & \omega^2\sqrt{2} & -\omega^2\sqrt{2} & 0 \end{pmatrix} \quad (12)$$

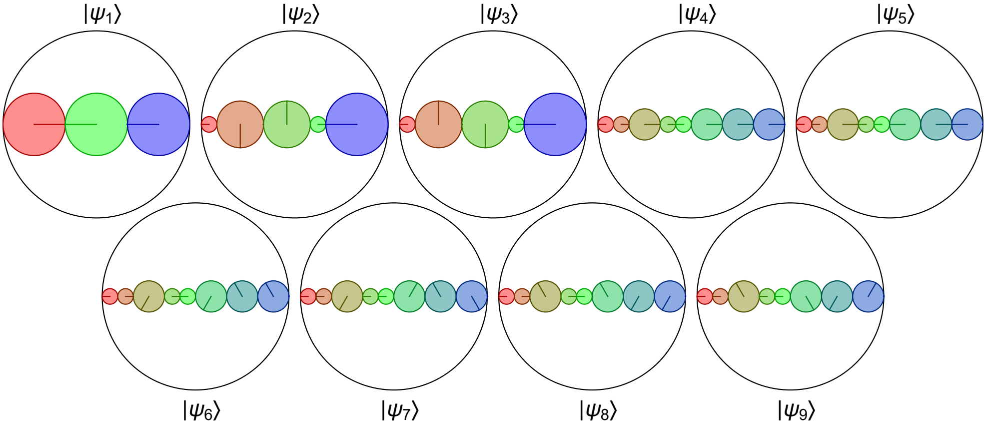
Maximally entangled counterpart

- For qutrits, it is also possible to construct a maximally entangled basis of a similar form, given by

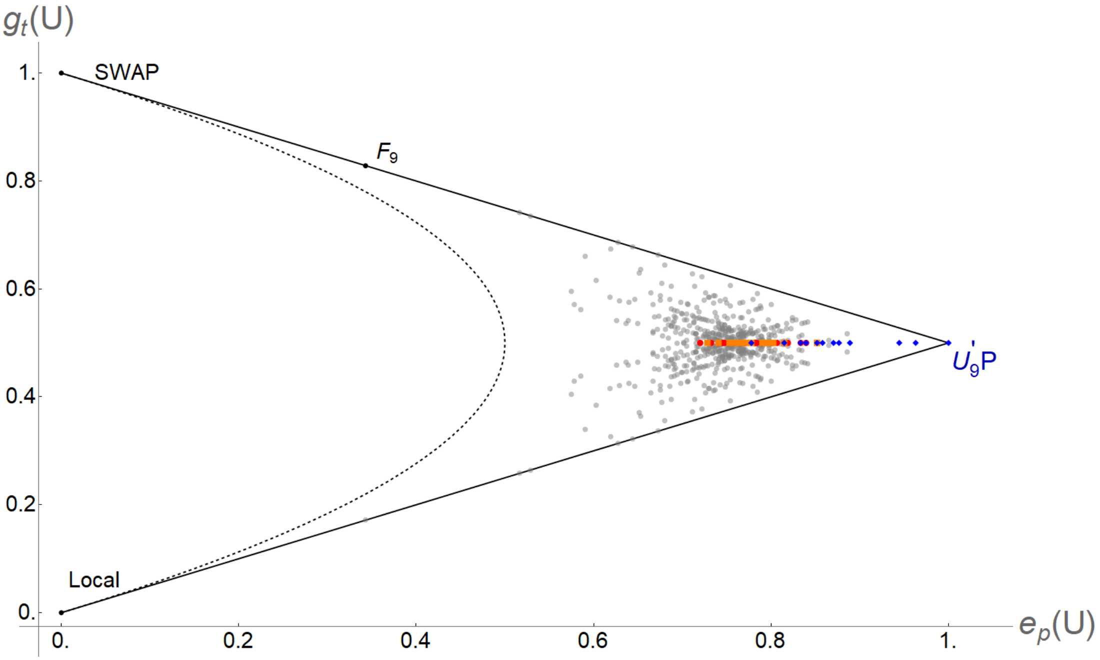


Maximally entangled counterpart

- For qutrits, it is also possible to construct a maximally entangled basis of a similar form, given by



- In terms of (e_p, g_t), it gives different points depending on the permutation of vectors, with some of them yielding the extreme point $(1, 1/2)$, corresponding to the 2-unitary matrix.


$g_t(U)$
1.
0.8
0.6
0.4
0.2
0.
SWAP
F_9
Local
$U_9'P$
0.
0.2
0.4
0.6
0.8
1.
$e_p(U)$

Tomographical power of the introduced bases

- From the point of view of a single-party reduction, the states can be viewed as noisy SIC-POVM.

$$Tr_A |\psi_i\rangle\langle\psi_i| = \rho_i = \frac{N\lambda - 1}{N-1} |i_0\rangle\langle i_0| + \frac{1-\lambda}{N-1}\mathbb{I}. \tag{12}$$

Tomographical power of the introduced bases

- From the point of view of a single-party reduction, the states can be viewed as noisy SIC-POVM.

$$Tr_A|\psi_i\rangle\langle\psi_i| = \rho_i = \frac{N\lambda - 1}{N-1}|i_0\rangle\langle i_0| + \frac{1-\lambda}{N-1}\mathbb{I}. \tag{12}$$

- This provides a tomographical power to such a measurement, with a linear reconstruction formula with $\tilde{\rho}_i = \frac{1}{N}\rho_i$ and $p_i = Tr(\sigma\tilde{\rho}_i)$,

$$\sigma = \frac{(N-1)^2 N(N+1)}{(\lambda N-1)^2}\sum_{i=1}^{N^2} p_i\tilde{\rho}_i - \frac{2\lambda + N^2 - \left(\lambda^2+1\right)N - 1}{(\lambda N-1)^2}\mathbb{I} \tag{13}$$

Tomographical power of the introduced bases

- From the point of view of a single-party reduction, the states can be viewed as noisy SIC-POVM.

$$Tr_A|\psi_i\rangle\langle\psi_i| = \rho_i = \frac{N\lambda - 1}{N-1}|i_0\rangle\langle i_0| + \frac{1-\lambda}{N-1}\mathbb{I}. \tag{12}$$

- This provides a tomographical power to such a measurement, with a linear reconstruction formula with $\tilde{\rho}_i = \frac{1}{N}\rho_i$ and $p_i = Tr(\sigma\tilde{\rho}_i)$,

$$\sigma = \frac{(N-1)^2 N(N+1)}{(\lambda N-1)^2}\sum_{i=1}^{N^2} p_i\tilde{\rho}_i - \frac{2\lambda + N^2 - (\lambda^2+1)N - 1}{(\lambda N-1)^2}\mathbb{I} \tag{13}$$

- This simplifies for qubits to

$$\sigma = 8\sum_{i=1}^{4} p_i\tilde{\rho}_i - \frac{3}{2}\mathbb{I} \tag{14}$$

Tomographical power of the introduced bases

- From the point of view of a single-party reduction, the states can be viewed as noisy SIC-POVM.

$$Tr_A |\psi_i\rangle\langle\psi_i| = \rho_i = \frac{N\lambda - 1}{N-1} |i_0\rangle\langle i_0| + \frac{1-\lambda}{N-1}\mathbb{I}. \tag{12}$$

- This provides a tomographical power to such a measurement, with a linear reconstruction formula with $\tilde{\rho}_i = \frac{1}{N}\rho_i$ and $p_i = Tr(\sigma\tilde{\rho}_i)$,

$$\sigma = \frac{(N-1)^2 N(N+1)}{(\lambda N-1)^2} \sum_{i=1}^{N^2} p_i \tilde{\rho}_i - \frac{2\lambda + N^2 - \left(\lambda^2+1\right) N - 1}{(\lambda N - 1)^2}\mathbb{I} \tag{13}$$

- This simplifies for qubits to

$$\sigma = 8\sum_{i=1}^{4} p_i\tilde{\rho}_i - \frac{3}{2}\mathbb{I} \tag{14}$$

- Errors from such scheme are approximately $4/3$ times the ones for SIC-POVM.

Gate implementation

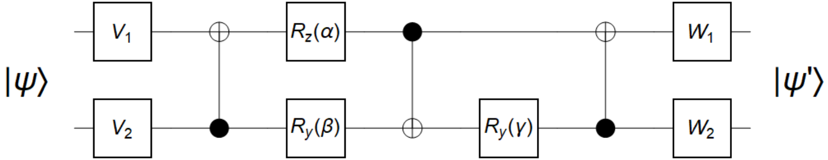
- In order to implement our scheme, we have resorted to decomposition of the gate into local gates V_i, W_i and the nonlocal gate U_c

$$U = (V_1 \otimes V_2) U_c (W_1 \otimes W_2) \tag{15}$$

- In order to implement our scheme, we have resorted to decomposition of the gate into local gates V_i, W_i and the nonlocal gate U_c

$$U = (V_1 \otimes V_2)U_c(W_1 \otimes W_2) \tag{15}$$

- U_c is expressable using 3 CNOT gates and 3 additional local rotations



with $\alpha = \frac{\pi}{4}$, $\beta = 0$ and $\gamma = -\frac{\pi}{2}$ for implementation of EJM.

The implementation on IBM machines

- We have tested the tomographical scheme using the open access IBM Quantum Experience quantum computers.

The implementation on IBM machines

- We have tested the tomographical scheme using the open access IBM Quantum Experience quantum computers.
- In particular, we have used IBM Melbourne 15-qubit and Oursense 5-qubit machines, using 8192 shots for each reconstruction.

The implementation on IBM machines

- We have tested the tomographical scheme using the open access IBM Quantum Experience quantum computers.
- In particular, we have used IBM Melbourne 15-qubit and Oursense 5-qubit machines, using 8192 shots for each reconstruction.
- As the target states of reconstruction, we have used the standard set of MUBs for a single qubit,

$$\begin{pmatrix}1\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix} \tag{16}$$

The implementation on IBM machines

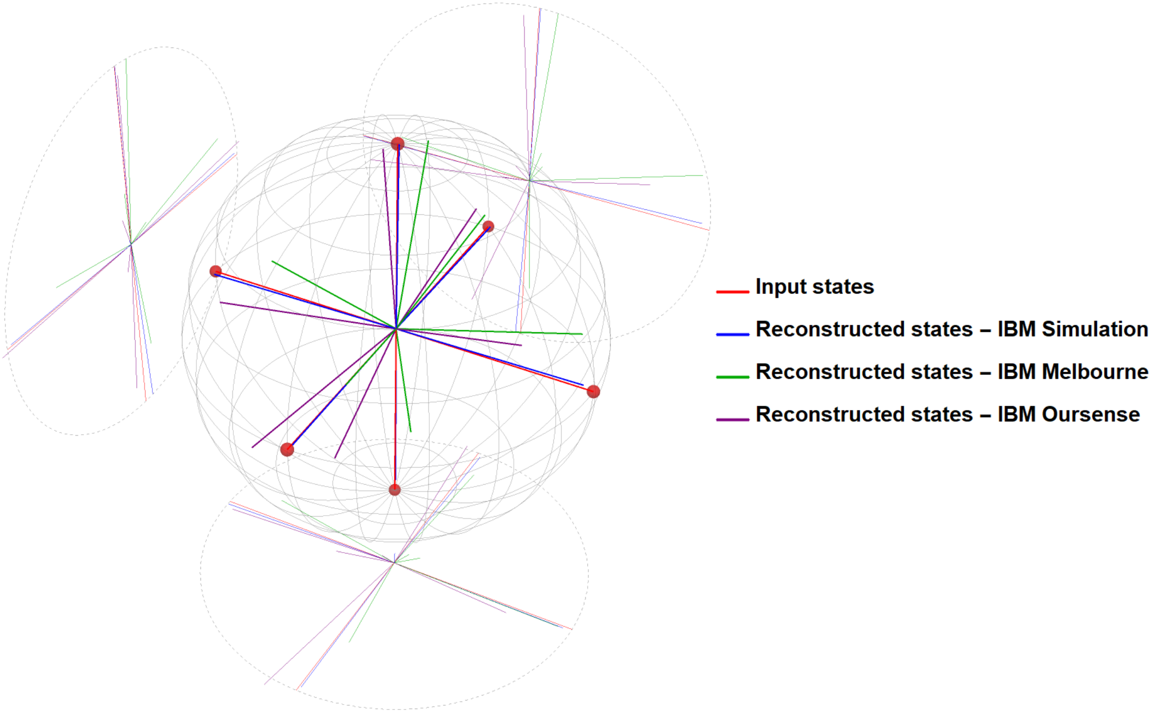
- We have tested the tomographical scheme using the open access IBM Quantum Experience quantum computers.
- In particular, we have used IBM Melbourne 15-qubit and Oursense 5-qubit machines, using 8192 shots for each reconstruction.
- As the target states of reconstruction, we have used the standard set of MUBs for a single qubit,

$$\begin{pmatrix}1\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix} \tag{16}$$

- As a measure of goodness of reconstruction, we have used a simple expression

$$\sqrt{\operatorname{Tr}\Delta\sigma^2} = \sqrt{\operatorname{Tr}(\sigma_{\text{theor}} - \sigma_{\exp})^2} \tag{17}$$

achieving $\sqrt{\operatorname{Tr}\Delta\sigma^2} \approx 0.027$ in simulations and $\sqrt{\operatorname{Tr}\Delta\sigma^2} \approx 0.177$ in real world implementation


Input states
Reconstructed states – IBM Simulation
Reconstructed states – IBM Melbourne
Reconstructed states – IBM Oursense

Summary

- We introduce a new class of bipartite basis in dimensions N^2, for which the pairwise single-sided distinguishability is optimal.

Summary

- We introduce a new class of bipartite basis in dimensions N^2, for which the pairwise single-sided distinguishability is optimal.
- We conjecture that their existence is directly related to the existence of SIC-POVM in dimension N.

- We introduce a new class of bipartite basis in dimensions N^2, for which the pairwise single-sided distinguishability is optimal.
- We conjecture that their existence is directly related to the existence of SIC-POVM in dimension N.
- For qubits, $N = 2$, we find our solution to be equivalent to EJM, introduced earlier by Gisin in the context of nonlocal quantum networks and show it is optimal in the (e_p, g_t) plane.

Summary

- We introduce a new class of bipartite basis in dimensions N^2, for which the pairwise single-sided distinguishability is optimal.
- We conjecture that their existence is directly related to the existence of SIC-POVM in dimension N.
- For qubits, $N = 2$, we find our solution to be equivalent to EJM, introduced earlier by Gisin in the context of nonlocal quantum networks and show it is optimal in the (e_p, g_t) plane.
- A maximally entangled member of the introduced family for qutrits, $N = 3$, proves to be closely related to a unitary operation with maximal entangling power, $e_p = 1$.

Summary

- We introduce a new class of bipartite basis in dimensions N^2, for which the pairwise single-sided distinguishability is optimal.
- We conjecture that their existence is directly related to the existence of SIC-POVM in dimension N.
- For qubits, $N = 2$, we find our solution to be equivalent to EJM, introduced earlier by Gisin in the context of nonlocal quantum networks and show it is optimal in the (e_p, g_t) plane.
- A maximally entangled member of the introduced family for qutrits, $N = 3$, proves to be closely related to a unitary operation with maximal entangling power, $e_p = 1$.
- We show that the introduced bases possess a tomographical power when considered from a point of view of a single party.

Summary

- We introduce a new class of bipartite basis in dimensions N^2, for which the pairwise single-sided distinguishability is optimal.
- We conjecture that their existence is directly related to the existence of SIC-POVM in dimension N.
- For qubits, $N = 2$, we find our solution to be equivalent to EJM, introduced earlier by Gisin in the context of nonlocal quantum networks and show it is optimal in the (e_p, g_t) plane.
- A maximally entangled member of the introduced family for qutrits, $N = 3$, proves to be closely related to a unitary operation with maximal entangling power, $e_p = 1$.
- We show that the introduced bases possess a tomographical power when considered from a point of view of a single party.
- We demonstrate this tomographic power by realizing the qubit scheme in IBM computers.

References

J. Cz. and Karol Życzkowski (2021)
Bipartite quantum measurements with optimalsingle-sided distinguishability
Quantum **5**, 442 (2021)

Armin Tavakoli, Nicolas Gisin and Cyril Branciard (2020)
Bilocal Bell inequalities violated by the quantum Elegant Joint Measurement
ArXiv: 2006.16694

Bhargavi Jonnadula, Prabha Mandayam, Karol Życzkowski, and Arul Lakshminarayan (2020)
Entanglement measures of bipartite quantum gates and their thermalization under arbitrary interaction strength
Phys. Rev. Research **2**, 043126

Jun Zhang, Jiri Vala, Shankar Sastry and K. Birgitta Whaley (2004)
Minimum construction of two-qubit quantum operations
Phys. Rev. Lett. **93**, 020502

Farrokh Vatan and Colin Williams (2004)
Optimal quantum circuits for general two-qubit gates
Phys. Rev. A **69**, 032315

Thank you for your attention!