



Generalized measurements on quantum devices

Filip Maciejewski, Michał Oszmaniec

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Outline

Introduction

Part I

"Simulating all quantum measurements using only projective measurements and postselection"

Michał Oszmaniec, FBM, and Zbigniew Puchała, Phys. Rev. A 100, 012351 (2019)

Part II

"Mitigation of readout noise in near-term quantum devices by classical post-processing based on detector tomography"

FBM, Zoltán Zimborás, and Michał Oszmaniec, Quantum 4, 257 (2020)

Summary + some current research topics





Generalized quantum measurements - POVMs

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 $M_i > 0$, $\sum_{i=1}^{n} M_i = 1$

5.1

 $M = (M_1, \dots, M_n)$

Projective measurements - PMs

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 $P = (P_1, o, o, P_n)$

 $\frac{1}{1} \frac{1}{1} \frac{1}$

Born's rule

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 $q(i|M,g) = Tr(g|M_i)$

PART I

PHYSICAL REVIEW A 100, 012351 (2019)



Simulating all quantum measurements using only projective measurements and postselection

Michał Oszmaniec,^{1,*} Filip B. Maciejewski,^{2,†} and Zbigniew Puchała^{3,4,‡} ¹Institute of Theoretical Physics and Astrophysics, National Quantum Information Centre, Faculty of Mathematics, Physics and Informatics, University of Gdansk, Wita Stwosza 57, 80-308 Gdańsk, Poland ²Faculty of Physics, University of Warsaw, Ludwika Pasteura 5, 02-093 Warszawa, Poland ³Institute of Theoretical and Applied Informatics, Polish Academy of Sciences, ulica Bałtycka 5, 44-100 Gliwice, Poland ⁴Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, ul. Łojasiewicza 11, 30-348 Kraków, Poland

(Received 15 August 2018; published 31 July 2019)

We report an alternative scheme for implementing generalized quantum measurements that does not require the usage of an auxiliary system. Our method utilizes solely (a) classical randomness and postprocessing, (b) projective measurements on a relevant quantum system, and (c) postselection on nonobserving certain outcomes. The scheme implements arbitrary quantum measurement in dimension d with the optimal success probability 1/d. We apply our results to bound the relative power of projective and generalized measurements for unambiguous state discrimination. Finally, we test our scheme experimentally on an IBM quantum processor. Interestingly, due to noise involved in the implementation of entangling gates, the quality with which our scheme implements generalized qubit measurements outperforms the standard construction using an auxiliary system.

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HTr(gøloxol Pi) = Tr(gMi) ancilla pM on extended space

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Answer – yes, but the protocol is probabilistic.

Simulation of POVMs by PMs We want to sample An - (M M M) $\mathcal{M} = (\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3)$ $T_r(gM_i)$

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(and we noticed that readout noise in those devices is terrible...)

PART II



Mitigation of readout noise in near-term quantum devices by classical post-processing based on detector tomography

Filip B. Maciejewski^{1,2,3}, Zoltán Zimborás^{4,5,6}, and Michał Oszmaniec^{2,3}

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 ⁶ Mathematical Institute. Budapest University of Technology and Economics, P.O.Box 91, H-1111, Budapest, Hungary

2020 We propose a simple scheme to reduce readout errors in experiments on quantum systems with finite number of measurement outcomes. Our method relies on performing classical postprocessing which is preceded by Quantum De-3 tector Tomography, i.e., the reconstruction of a Positive-Operator Valued Measure (POVM) describing the given quantum measurement device. If the measurement device is affected only by an invertible classical noise, it is possible to correct the outcome statistics of future experiments performed on the same device. To support the practical applicability of this scheme for nearterm quantum devices, we characterize measurements implemented in IBM's and Rigetti's quantum processors. We find that for these devices, based on superconducting transmon qubits, clas-

sical noise is indeed the dominant source of readout errors. Moreover, we analyze the influ-00 ence of the presence of coherent errors and finite statistics on the performance of our error-5 mitigation procedure. Applying our scheme on 00 0 the IBM's 5-qubit device, we observe a significant improvement of the results of a number of single- and two-qubit tasks including Quantum State Tomography (QST), Quantum Process Tomography (QPT), the implementation of nonprojective measurements, and certain quantum algorithms (Grover's search and the Bernstein-Vazirani algorithm). Finally, we present results ar. showing improvement for the implementation of certain probability distributions in the case of five qubits.



Figure 1: Pictorial representation of our error-mitigation procedure. (i) In the first stage, one performs the tomography of a noisy detector $\mathbf{M}^{\text{noisy}}$ (red semicircle). (ii) In the next stage, when measuring an arbitrary quantum state ρ , one employs a post-processing procedure on the measured statistics through the application of Λ^{-1} , the inverse of a stochastic noise map obtained in the QDT. This gives access to the statistics that would have been obtained in an ideal detector $\mathbf{M}^{\text{ideal}}$ (green semicircle).

of delicate quantum states with unprecedented precision [1]. Due to the advent of quantum cloud services (IBM [2, 3], Rigetti [4], DWave [5]), any researcher has a possibility to perform experiments on actual quantum devices. However, if one really hopes for utilizing such near-term devices for real-life applications such as quantum computation [6], quantum simulations [7] or generating random numbers [8], experimental imperfections must be taken into account. Hence, to properly characterize noise occurring in the devices and to develop error correction and mitigation schemes that may help to fight it have become tasks of fundamental importance [9, 10, 11, 12, 13, 14, 15]. In the present work, we address this problem for the

Quantum 4 257 (2020)



Previously, we wanted to implement POVMs to achieve some goal.
Now:

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Now:

We want to estimate this

Tr(gP)

Previously, we wanted to implement POVMs to achieve some goal.
 Now:

We want to estimate this This The Pile NOISE

Previously, we wanted to implement POVMs to achieve some goal.
Now:

Instead, we estimate

Tr (e Mi)

< this

However, if the noise is classical: D NOISE M We want to estimate Instead, we want to estimate Instead, we are this This The this this the Instead, we estimate < this

We want to estimate this The Pilling

However, if the noise is classical:
 NOISE

Instead, we estimate

this


Mitigation of classical noise

Hence, on the level of probability vectors:



Mitigation of classical noise

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QP CLASSICAL A QP

A⁻¹ ← MITIGATION

We analyzed the effects of non-classical noise:

• We analyzed the effects of non-classical noise: M = M + M + M

We analyzed the effects of non-classical noise:

COHERENT PART

 $M = (\Lambda \vec{P})$

CLASSICAL DART

We analyzed the effects of non-classical noise:

 $(\int \vec{p})$

STATISTICS

-

COHERENT

ME

CLASSICAL DART

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STATISTICS

COHERENT

 $M = (\Lambda \vec{P})$

 $q_{\tilde{M}} = \Lambda q_{\tilde{R}}$

CLASSICAL DART

We analyzed the effects of non-classical noise:

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COHERENT

• We analyzed the effects of non-classical noise: CLASSICAL QM = (1QP) + (3QP) + (3

COHERENT PART

We analyzed the effects of non-classical noise:

CLASSICAL QM = 10,7 DART MITIGATION

COHERENT PART

We analyzed the effects of non-classical noise: CLASSICAL QM = 197 + DART

MITIGATION

COHERENT PART

We analyzed the effects of non-classical noise:

9 - M_

MITIGATION

197

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DENOISED STATISTICS

CLASSICAL

• And the effects of finite-size statistics:

• And the effects of finite-size statistics:

PROBABILITIES

And the effects of finite-size statistics:

9, ESTIMATION

PROBABILITIES

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PROBABILITIES

FREQUENCIES

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- I have the results in additional slides!

Summary + some current research topics



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- We validated classical noise model on Rigetti's and IBM's device.
- We experimentally benchmarked our mitigation procedure in various quantum information protocols on up to 5 qubits.

+ some current research topics

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Additional slides

Experimental results - noise validation

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Experimental results - noise validation

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And we benchmarked our mitigation procedure on IBM's device:

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STAIE TOMOGRAPHY





FIVE QUBIT

PROBABILITY DISTRIBUTIONS DISTRIBUTIONS



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Name	Standard	Corrected	α
Uniform	0.110 ± 0.006	0.100 ± 0.007	0
NOT	0.66 ± 0.02	0 ± 0	0.36 ± 0.09
Mixed	0.196 ± 0.006	0.031 ± 0.008	0.019 ± 0.005

(a) Without accounting for correlations.

Name	Corrected	α
Uniform		÷
NOT	\pm	÷
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(b) Accounting for correlations for **one pair**.

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Name	Corrected	α
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