# Quantum non-locality in Networks

**Nicolas Gisin**

**GAP, Geneva University**

**How to test quantumness is networks ?**

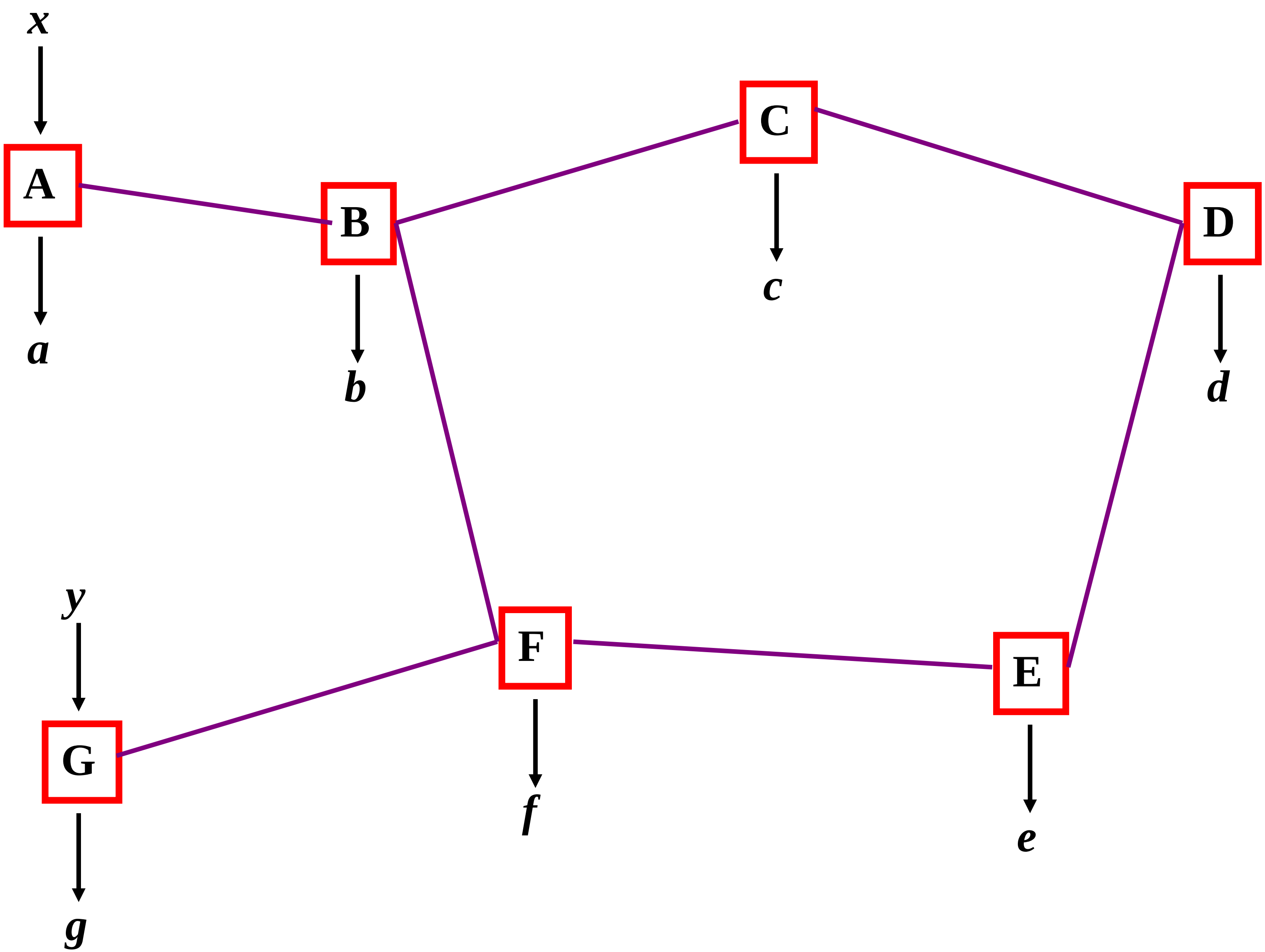
1. For 2-party scenarios it doesn’t matter whether the shared randomness is God given or carried by particles from common sources.
2. In realistic networks there are many independent sources.
3. The “God-given” view is the common view, but not the realistic view.

Bellevu

Bernex

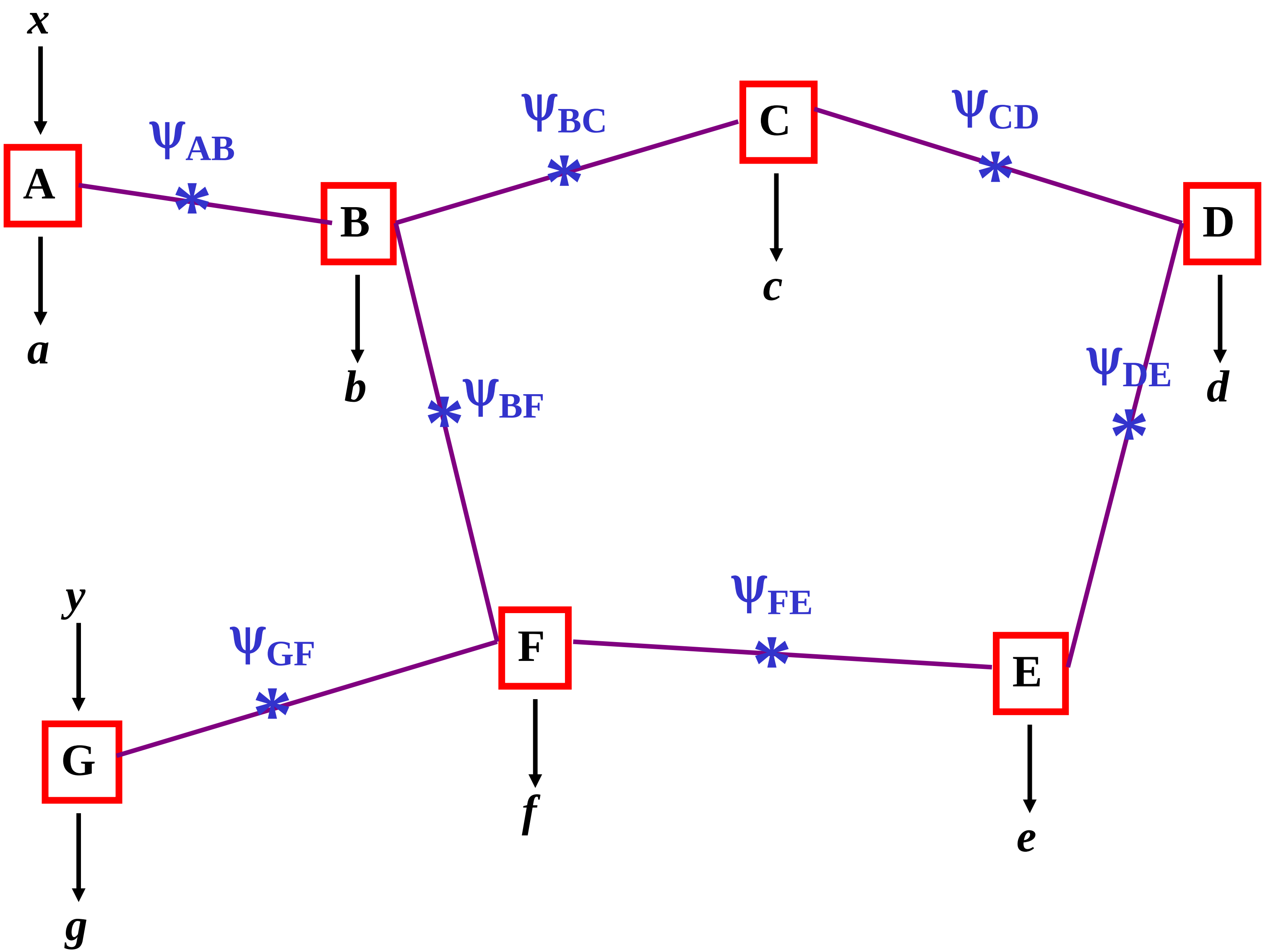
# Example of a Networks





# Quantum Networks

**With <u>independent</u> quantum states $\psi_{ij}$**

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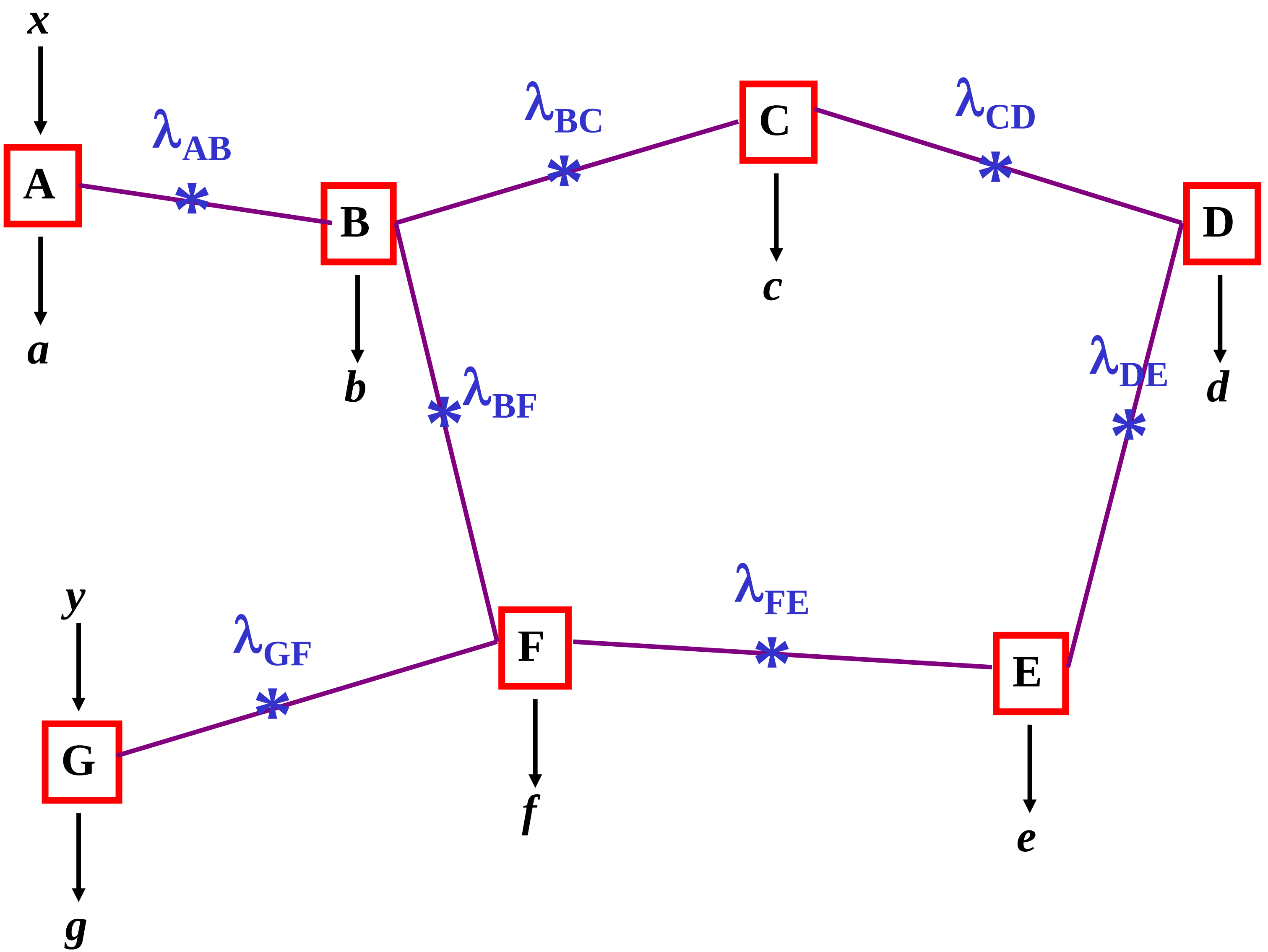
# Classical Networks

**With <u>independent</u> shared randomness $\lambda_{ij}$**

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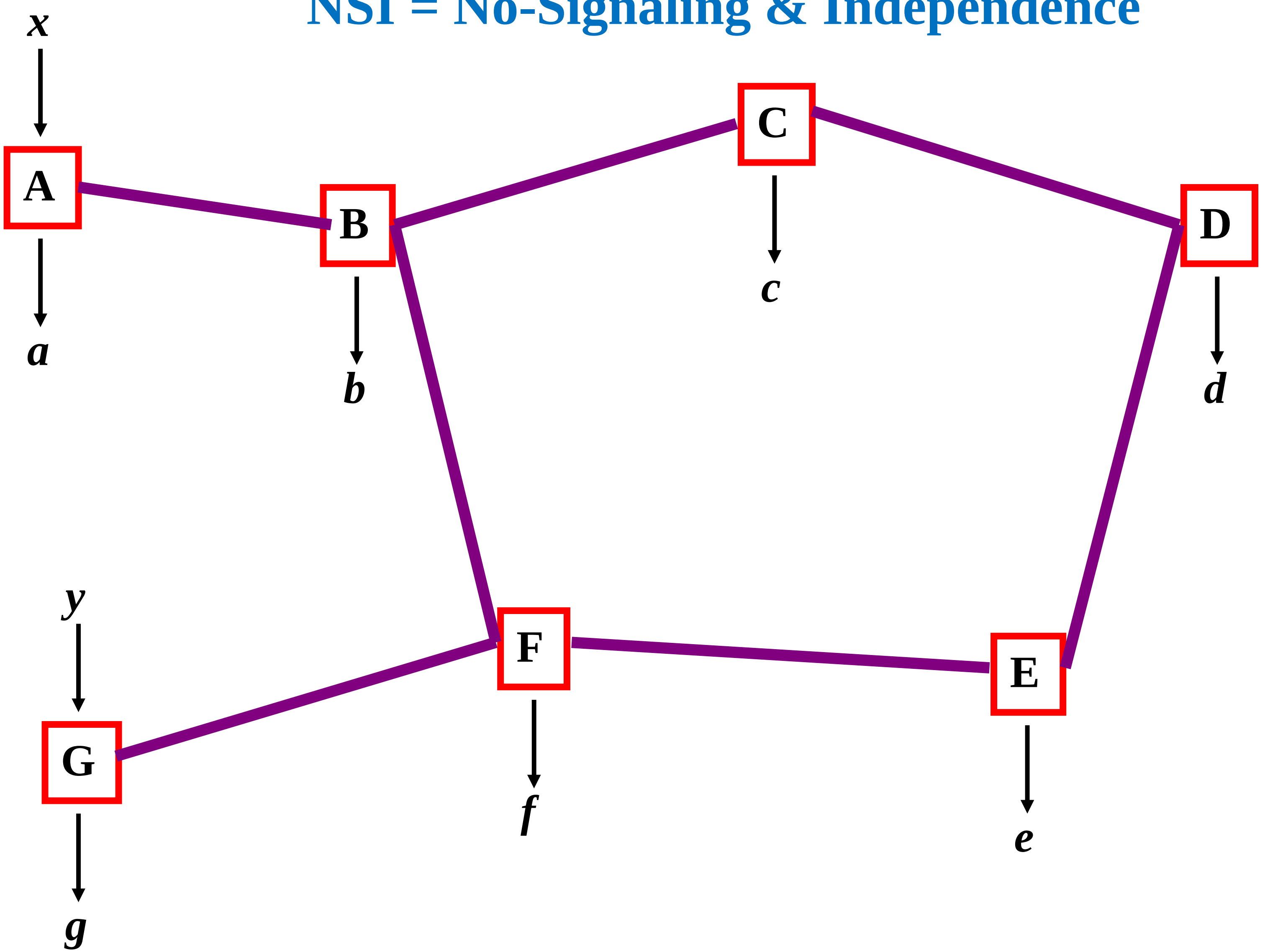
# Non-Local Networks

**With non-local boxes satisfying the NSI principle:**
**NSI = No-Signaling & Independence**

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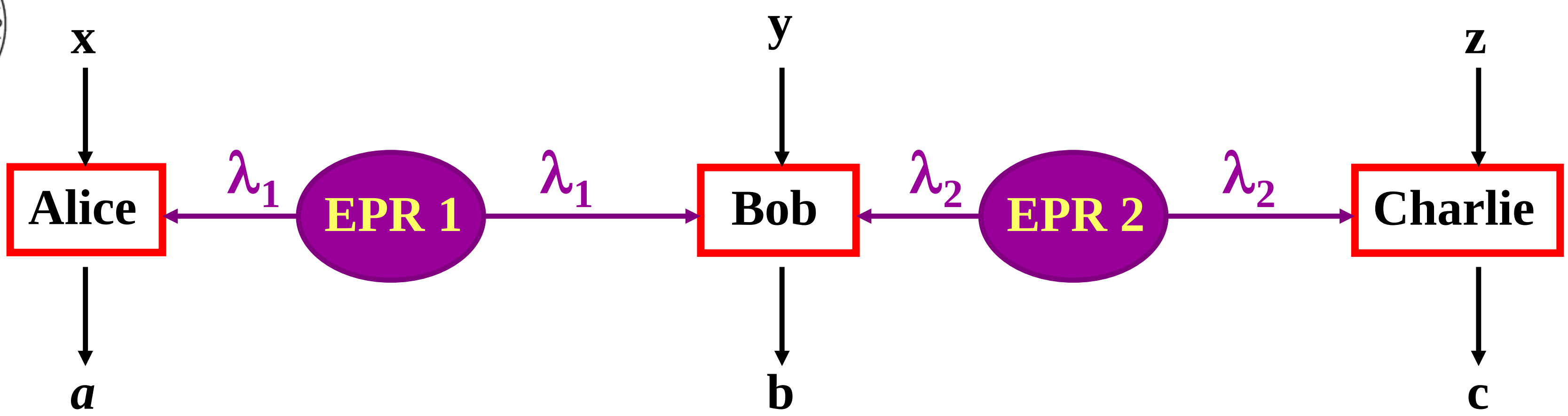
# Bi-locality

Branciard, NG, Pironio
PRL 104, 170401-1/4, 2010




$\Rightarrow$ **P(a,b,c|x,y,z)**

**Independent locality (bi-locality) :** $P(\lambda 1,\lambda 2)=P(\lambda_1)\cdot P(\lambda_2)$

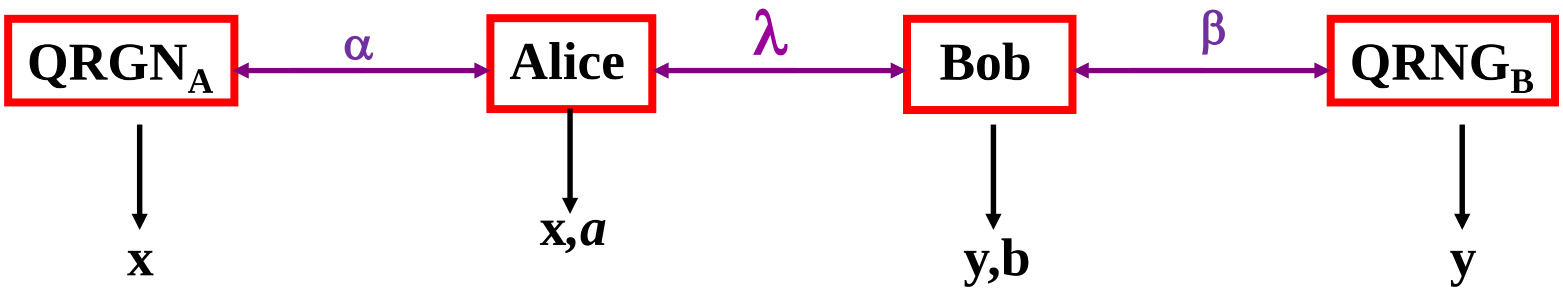
$$P_{biloc.}(a,b,c|x,y,z,\lambda_1,\lambda_2) = p(a|x, \lambda_1) \cdot p(b|y, \lambda_1, \lambda_2) \cdot p(c|z, \lambda_2)$$

$\Rightarrow$ **??? Bell-like inequalities ???**

**The NSI assumption when the sources EPR1 and EPR2 are independent is as natural as Bell's locality assumption.**

**It is implicitly assumed in all Bell test with Quantum Random Number Generators (QRNG).**

# Bell inequality without inputs

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# Bi-locality

Branciard, NG, Pironio
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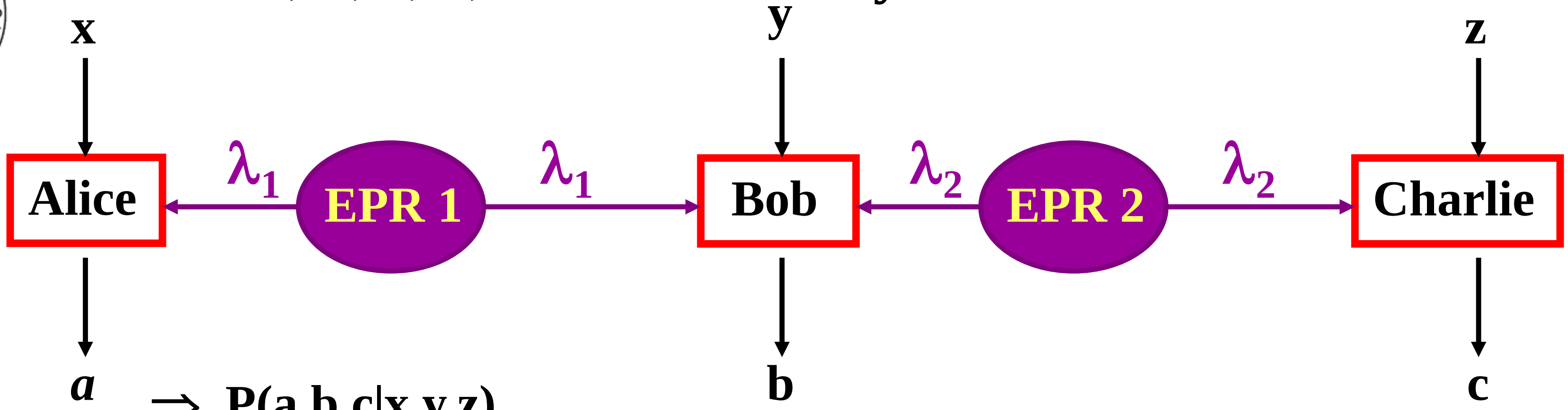
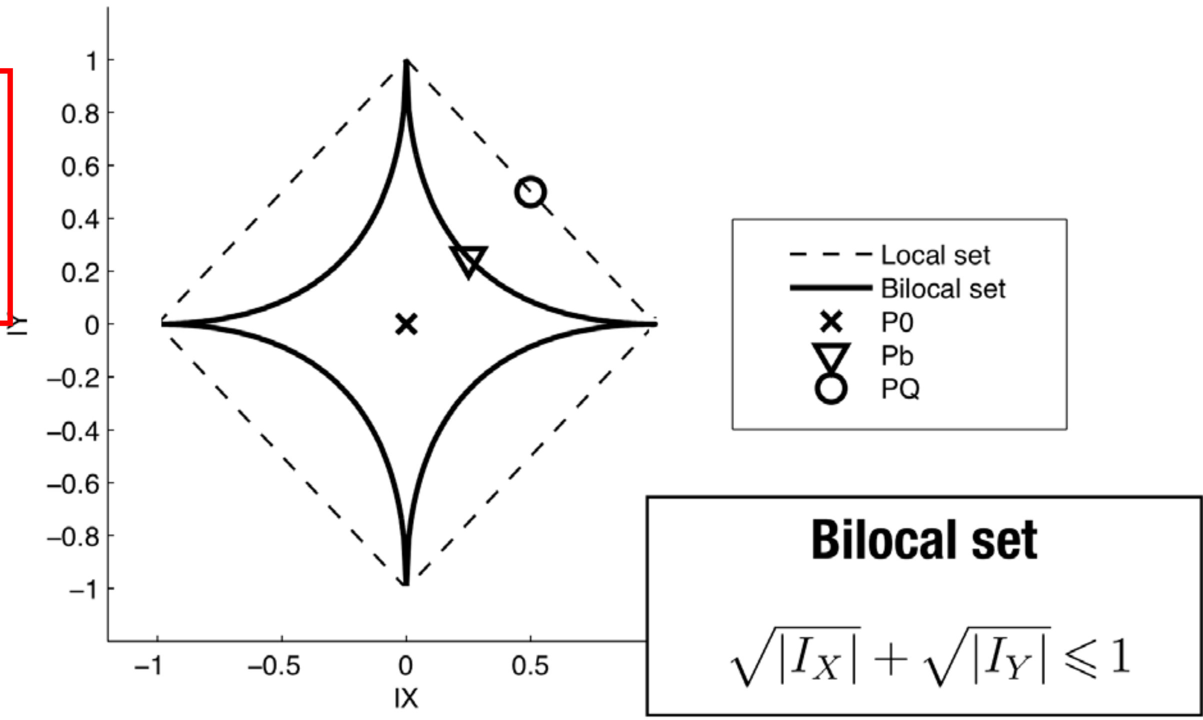
***a***, **b, c, x, z are all binary**




$\Rightarrow$ **P(a,b,c|x,y,z)**

**The set of bilocal $p_{biloc}(a,b,c\,|\,x,y,z)$ is not convex:**

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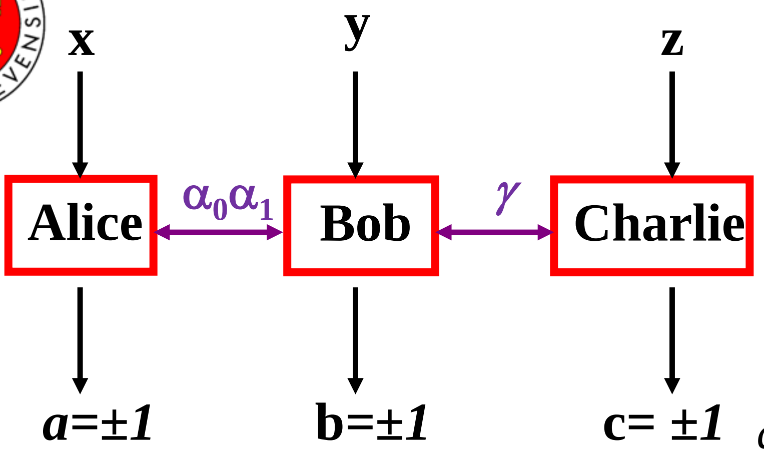
# Derivation of the bilocal inequality



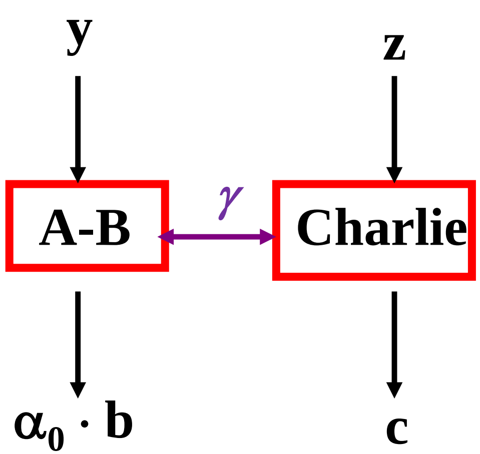



$$I=\frac{1}{4}\sum_{\substack{x,z=0,1\\ y=0}}(-1)^{xy+yz}\langle a_x b_0 c_z\rangle=\frac{1}{2}\sum_{z=0,1}\left\langle\frac{a_0+a_1}{2}b_0 c_z\right\rangle$$

$$J=\frac{1}{4}\sum_{\substack{x,z=0,1\\ y=1}}(-1)^{xy+yz}\langle a_x b_1 c_z\rangle=\frac{1}{2}\sum_{z=0,1}(-1)^z\left\langle\frac{a_0-a_1}{2}b_1 c_z\right\rangle$$

$$q\equiv prob(\alpha_0=\alpha_1)\Rightarrow I=\frac{1}{2}\sum_z\langle\alpha_0 b_0\cdot c_z\rangle\cdot q$$

$$J=\frac{1}{2}\sum_z(-1)^z\langle\alpha_0 b_1\cdot c_z\rangle\cdot(1-q)$$




$$CHSH\Rightarrow\left\langle\alpha_0 b_0\cdot\frac{c_0+c_1}{2}\right\rangle+\left\langle\alpha_0 b_1\cdot\frac{c_0-c_1}{2}\right\rangle\le 1$$

$$\exists\, q,\quad \frac{I}{q}+\frac{J}{1-q}\le 1\quad\Rightarrow\quad p(a,b,c\,|\,x,y,z)\ \text{is bilocal}$$

$$\Rightarrow\quad\sqrt{|I|}+\sqrt{|J|}\le 1$$

**All pairs of pure states violate this bi-locality inequality.** PRA **96**, 020304(R) (2017)

**Too similar to CHSH !**

# Bi-locality

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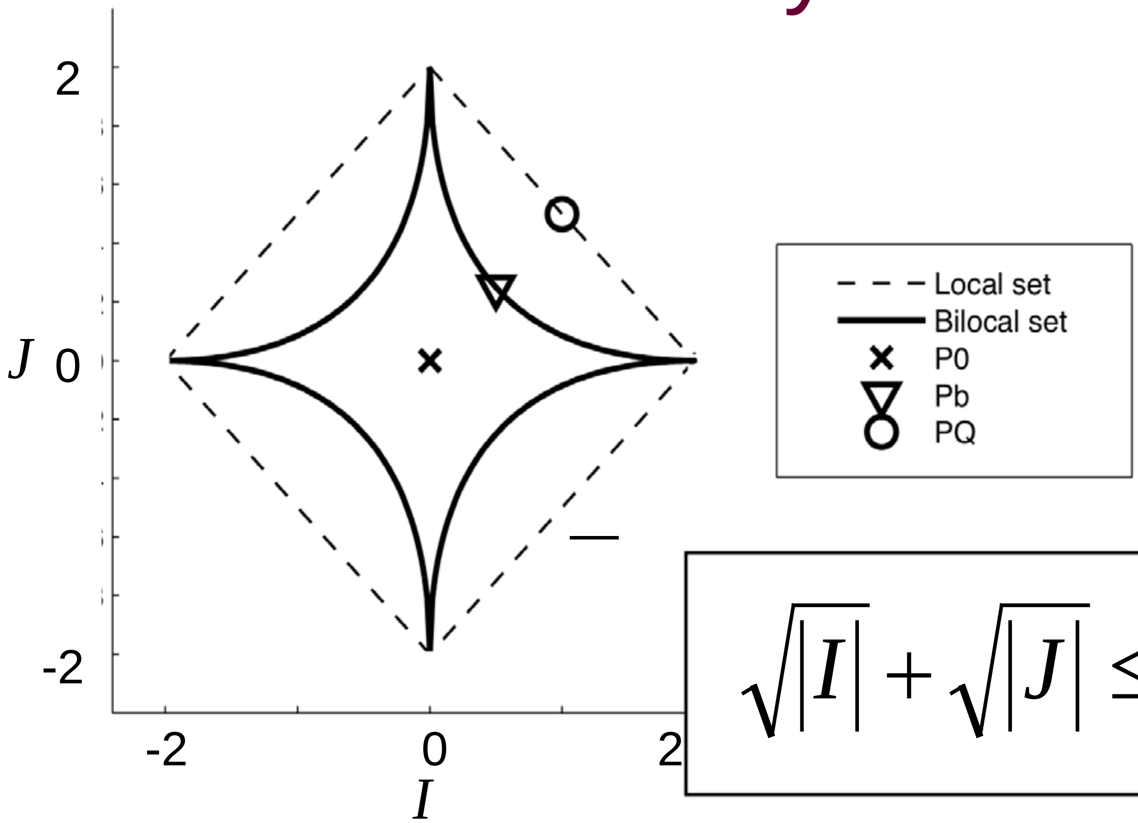
Branciard, NG, Pironio
PRL 104, 170401-1/4, 2010

Rosset +Branciard et al.
PRL 116, 010403, 2016




$$I = \frac{1}{4}\sum_{x,z}\langle a_x b_0 c_z\rangle$$

$$J = \frac{1}{4}\sum_{x,z}(-1)^{x+z}\langle a_x b_1 c_z\rangle$$

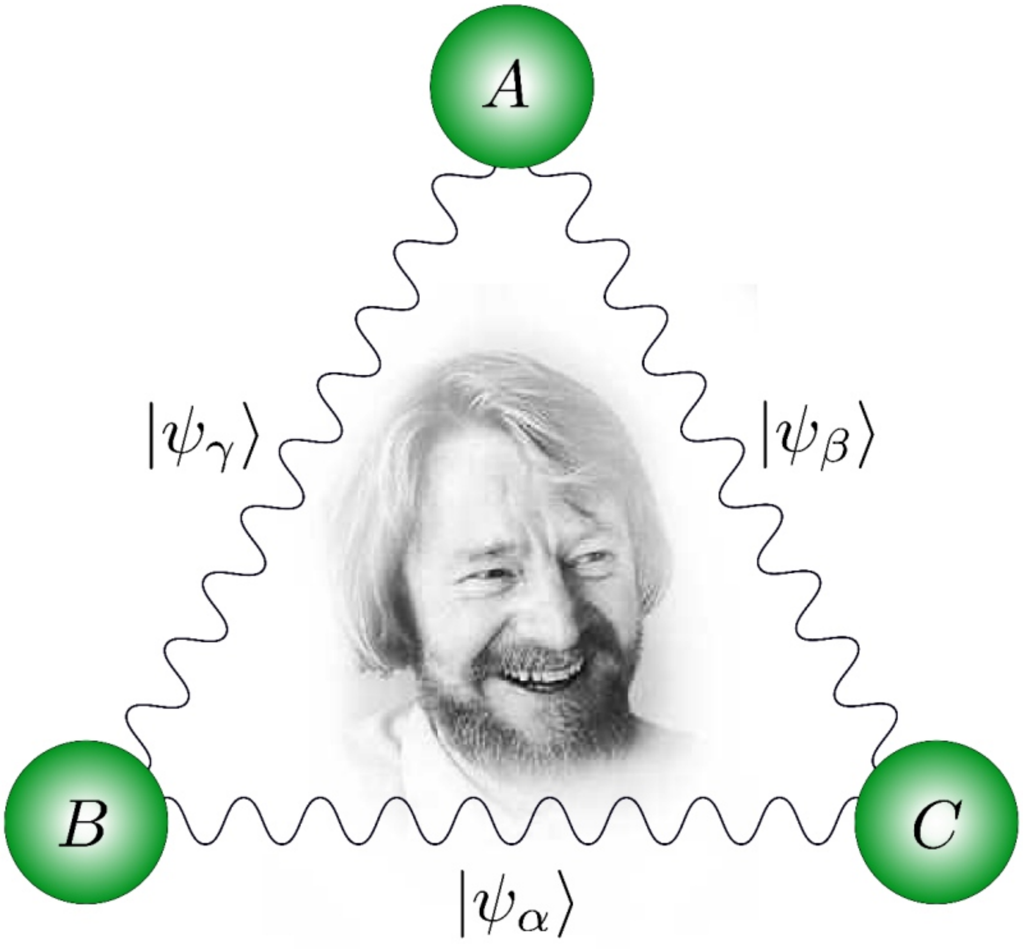
$$\sqrt{|I|} + \sqrt{|J|} \leq 1$$

**Critical visibility $V_{th}$=1/2, significantly lower than for CHSH, hence it is easier to experimentally demonstrate quantumness.**

**But the critical visibility per singlet is still 1/√2. Disappointing !**

**Challenge 1: find a bi-locality scenario with critical visibility per singlet < 1/√2**

# Quantum Networks with a Loop

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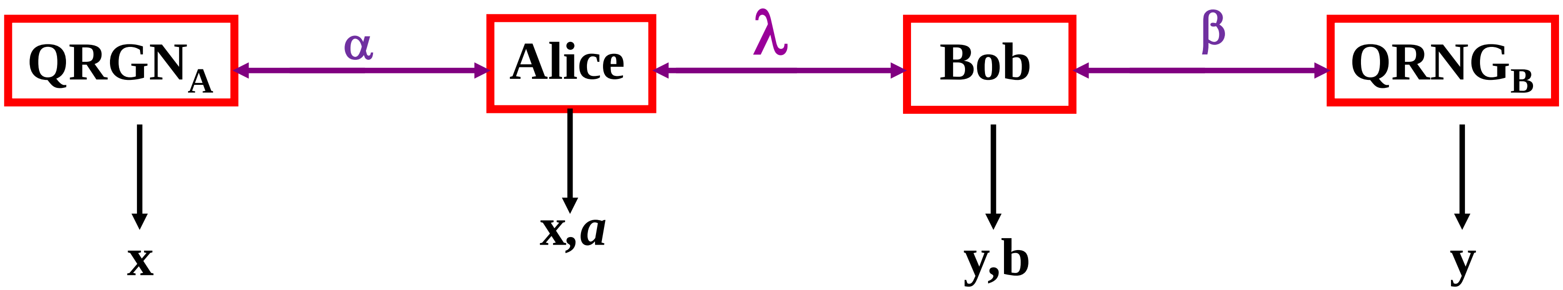
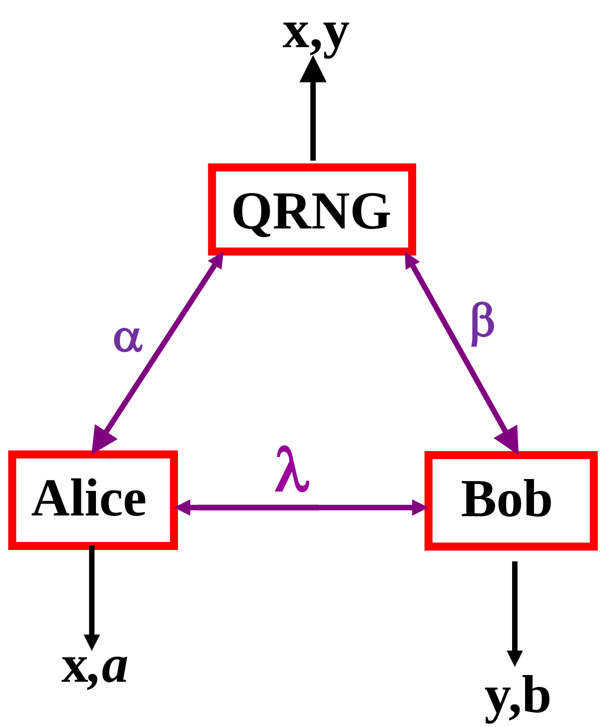
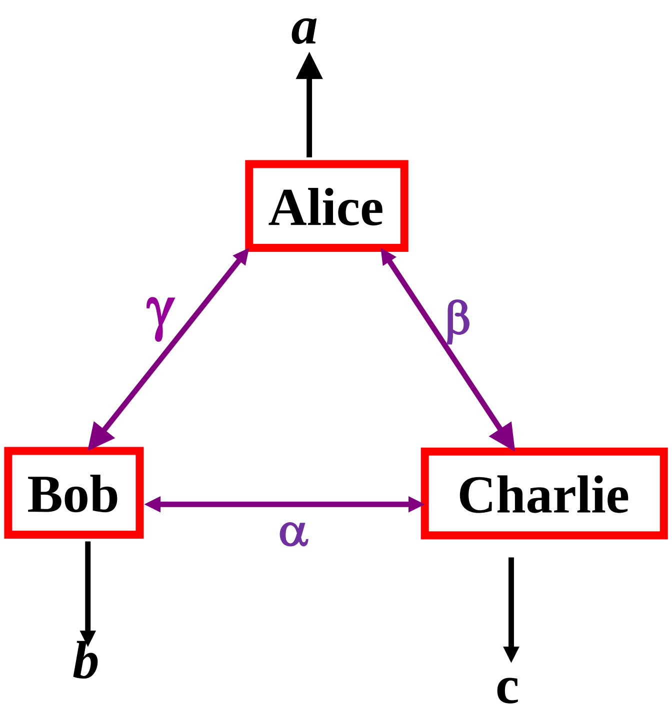
# Bell inequality without inputs




T. Fritz, NJP 14, 103001 (2012)

# The – genuine – Triangle

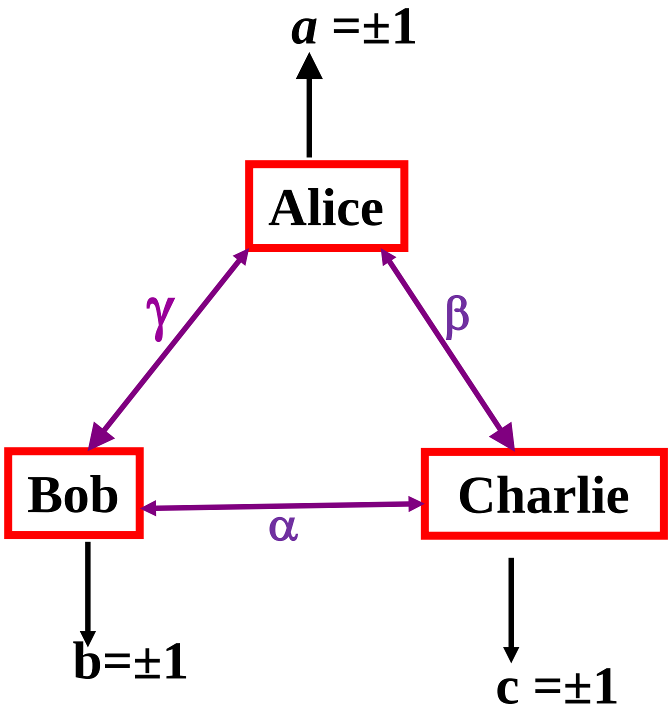



- **Is there a quantum case (i.e. feasible with quantum states and measurements) which is not 3-local ?**
- **Can the local variables be discrete? How many symbols?**

**Example: α,β,γ uniformly random in [0,1]; *a*=(β≥γ), b=(γ≥α), c=(α≥β)**
**⇒ p(a=b=c)=0 and all other p(a,b,c)=1/6**
**Impossible to reproduce with finite alphabets and identical output fcts.**
**But feasible with trits and different output fcts. QIC 18, 910-926 (2018)**

# The Triangle with binary outcomes

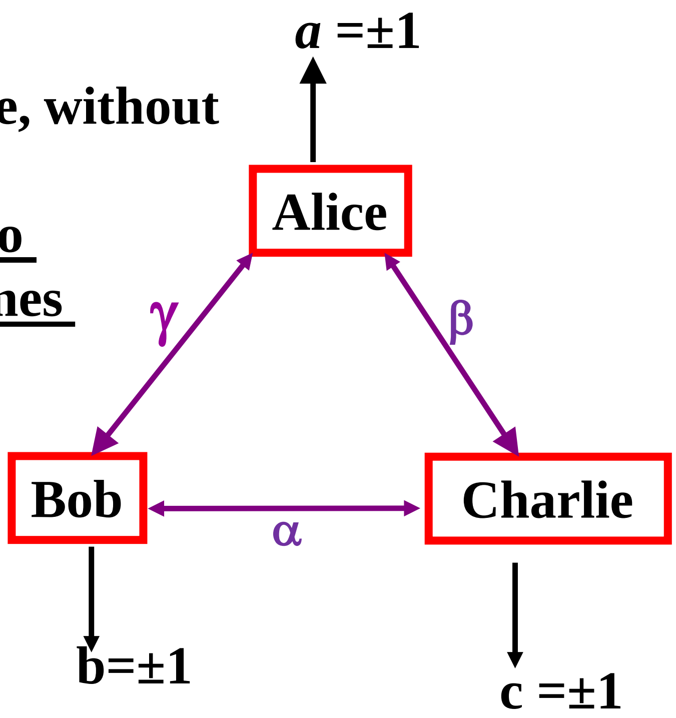
P(a,b,c)




- Denis Rosset et al.: QIC 18, 910-926 (2018)
  ⇒ **Alphabets of 6 letters suffices.**
- **How to characterize the p(a,b,c) that satisfy the NSI principle ?**
  **NSI = No-Signaling and Independence.**

# The Triangle with binary outcomes

**A 6 qubit toy universe, without input. How to prove quantumness? <u>How to prove that the outcomes contain some randomness?</u>**



# Quantum Technology

# From Physics to Technology

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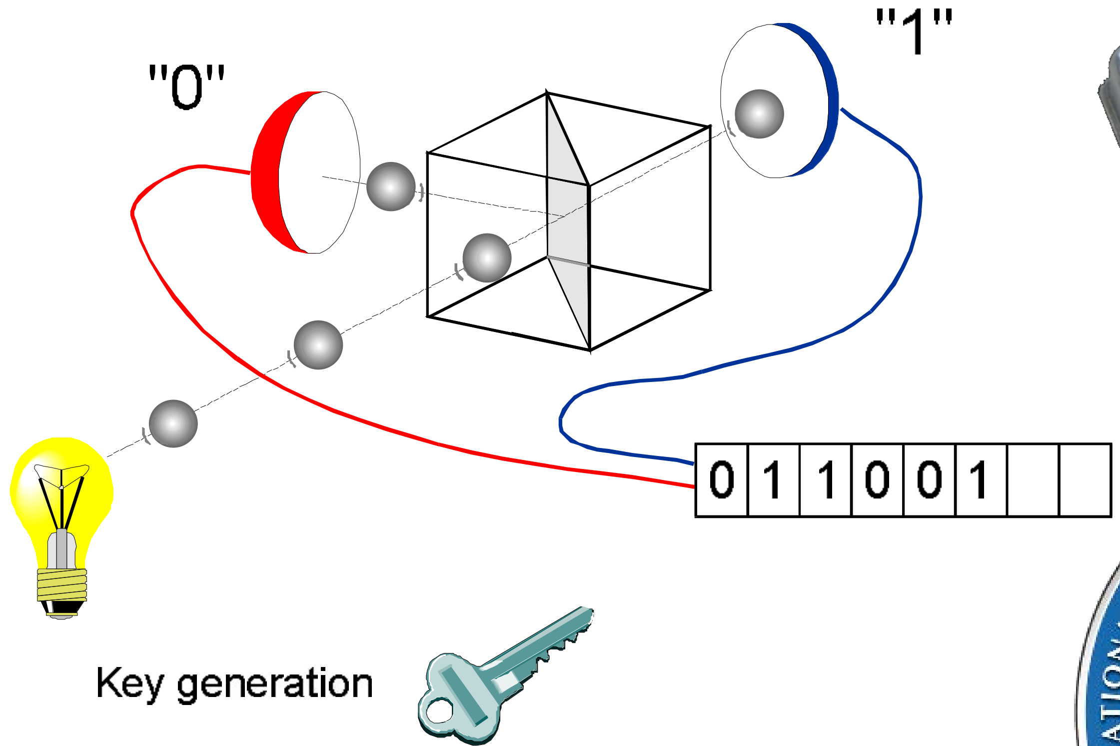
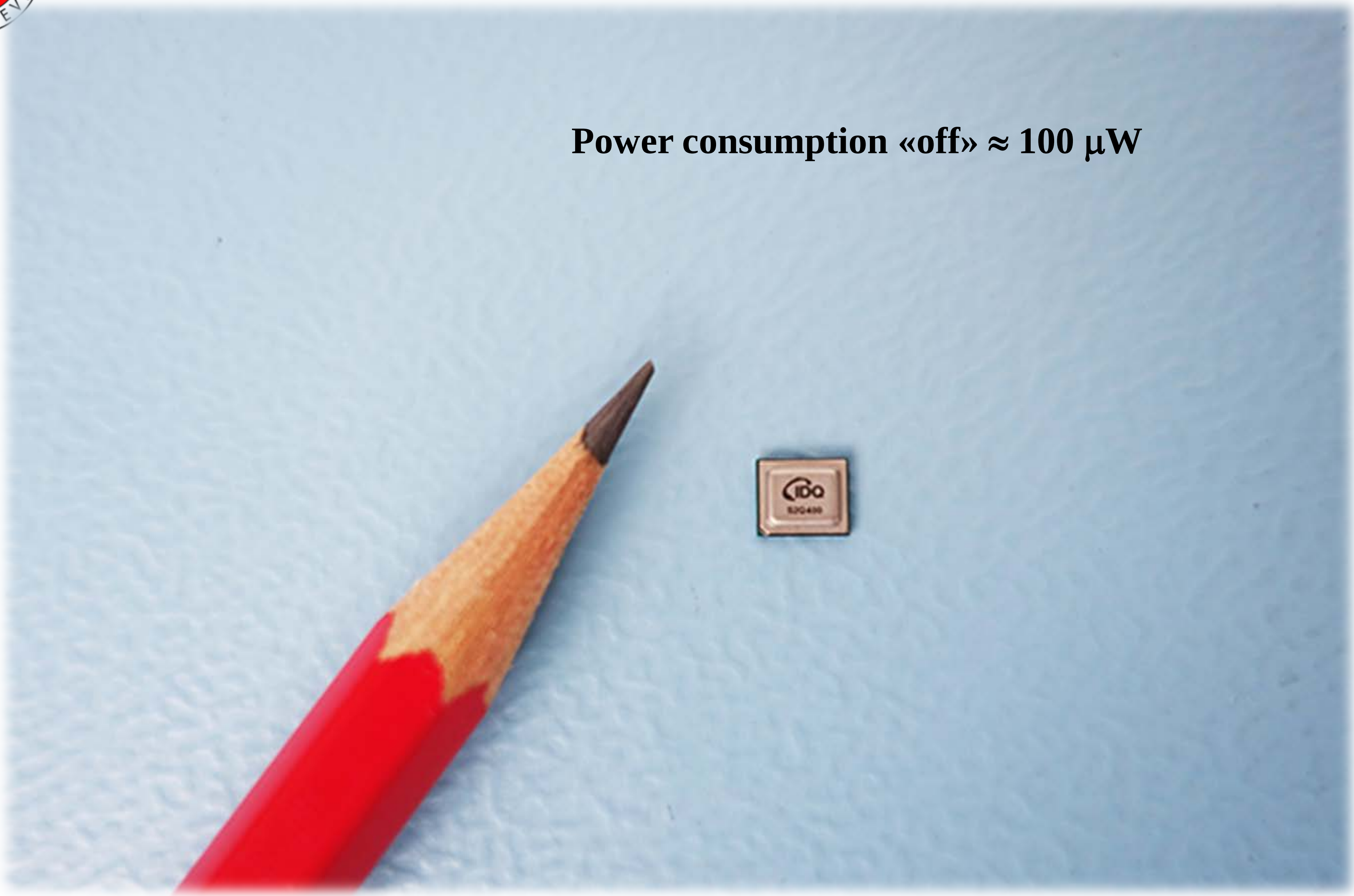




**certification**

- **A conceptually simple entropy source**
- **Only quantum physics offers fundamental randomness**
- **Easy to pindown the origine of the randomness**
- **A practical random number generator**

# mm-scale QRNG-chip



**Power consumption «off» ≈ 100 μW**

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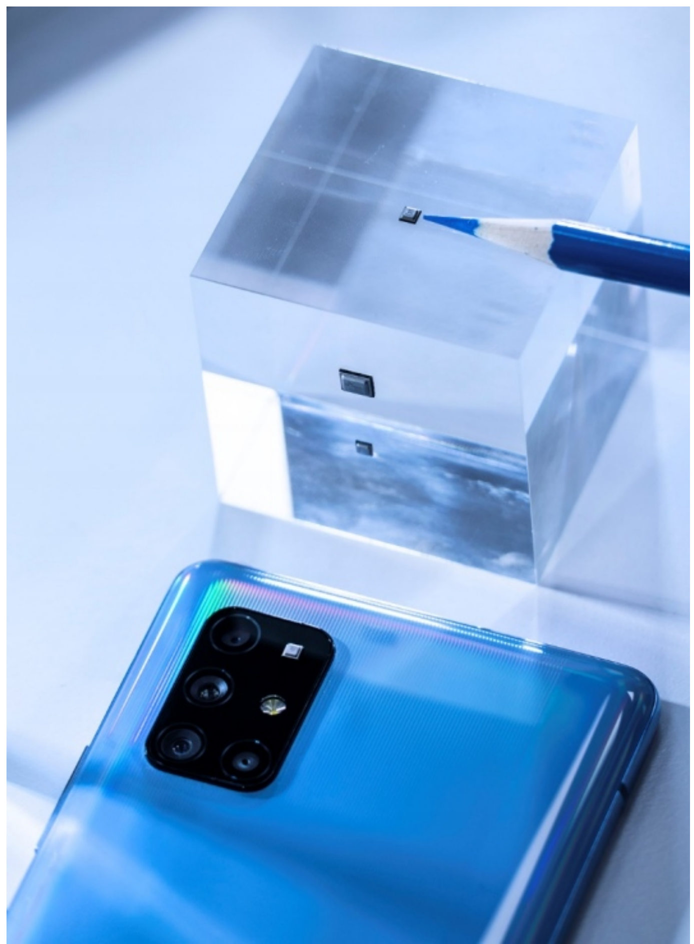
# Back to Quantum Entanglement Science

SKT 5GX

QUANTUM

Secured by Swiss Quantum



# The Triangle with binary outcomes

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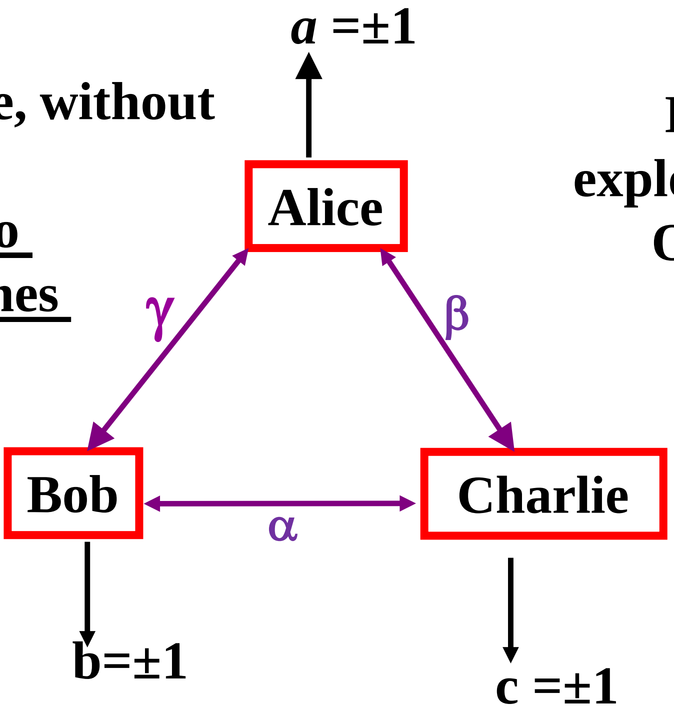
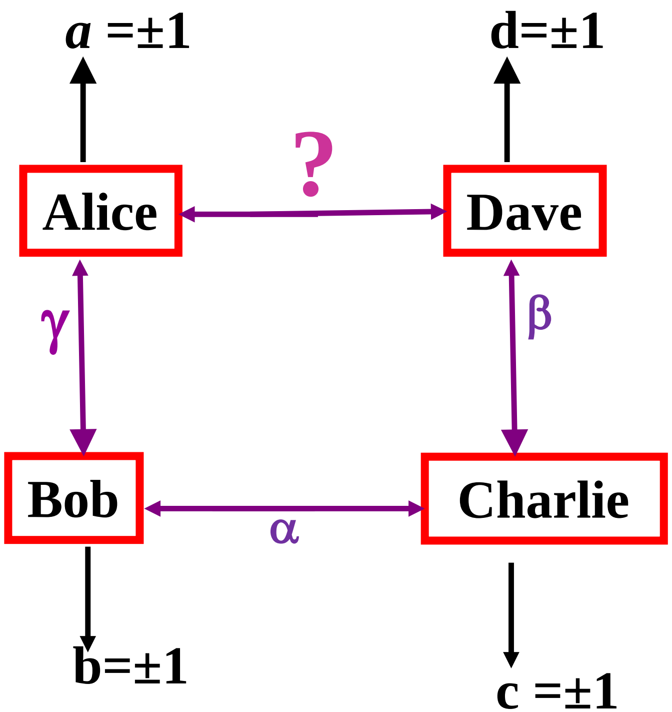



**How to formalize and exploit the NSI principle? One has only access to P(a,b,c).**

**There are no inputs. However, Alice can decide not to perform any measurement, or to change the local topology, ⇒ that should not affect the B-C correlation: no-signaling.**

# The Triangle with binary outcomes




**There are no inputs. However, Alice can decide not to perform any measurement, or to change the local topology, ⇒ that should not affect the B-C correlation: no-signaling.**

**Alice and Charlie are independent. Similarly, Dave and Bob are independent.**

**NSI**

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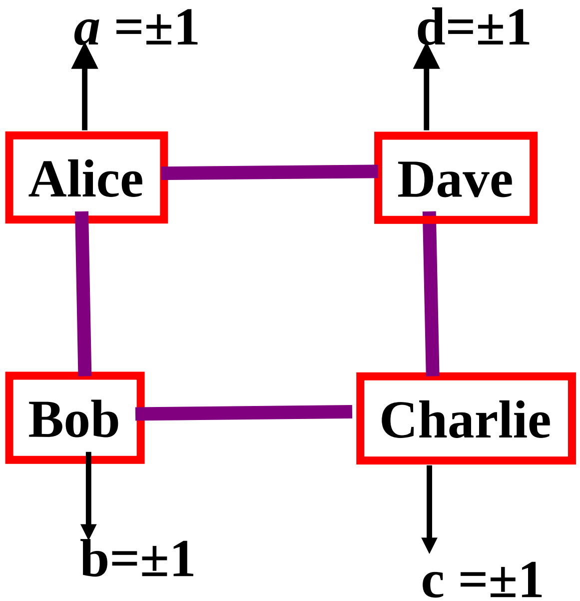
# NSI for the triangle with symmetric non-local boxes inflated to a square




**E1 = 0 (locally random)**

$E_{AB}=E_{BC}=E_{CD}=E_{DA} = E2$

**No-signaling ⇒**

**E2 square = E2 triangle**

$E_{ABC}=E_{BCD}=E_{CDA}=E_{DAB} = E3$

$E_{AC}=E_{BD}=0$ **Independence**

$$P(a,b,c,d)=\frac{1}{16}\begin{pmatrix}1+(ab+bc+cd+da)\cdot E2\\ +(abc+bcd+cda+dab)\cdot E3\\ +abcd\cdot E4\end{pmatrix}$$

$$P(+-+-)=\frac{1}{16}(1-4\cdot E2+E4)\geq 0$$

$$\Rightarrow 1\geq E4\geq 4\cdot E2-1$$

$$\Rightarrow E2\leq\frac{1}{2}$$

**arXiv:1906.06495**
**Nature Commun. in press**

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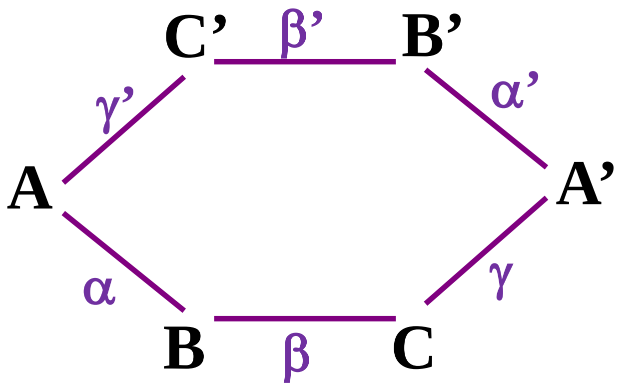
# Inflation to Hexagon

E.Wolfe et al.,
arXiv:1609.00672

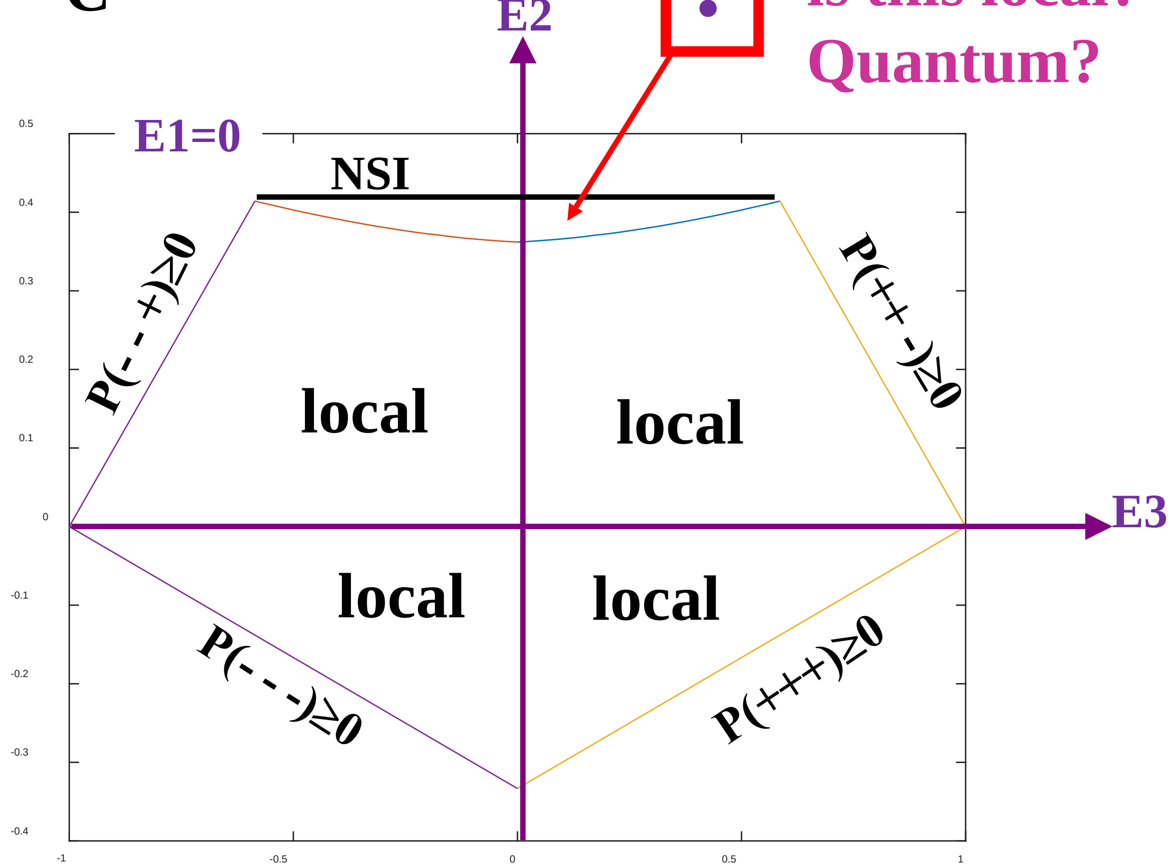



$$E1 = 0 \quad \Rightarrow \quad E2 \le \sqrt{2} - 1$$

?

**Challenge 2:**
**is this local?**
**Quantum?**

**Triangle :**

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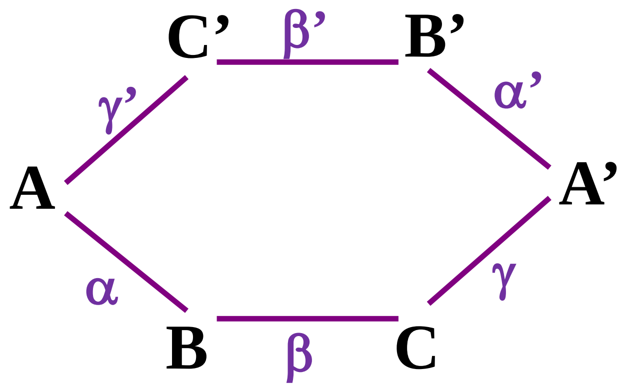
arXiv:1906.06495
Nature Commun. in press

# With no symmetry assumption




**Challenge 3:** find a triangle NSI-inequality that involves the 3-party correlator.

**NSI inequalities for the triangle:**

$$E_A=E_B=E_C=0 \Rightarrow (1+E_{AB})^2+(1+E_{BC})^2+(1+E_{CA})^2 \leq 6$$

$$(1+2|E_AE_B|+E_{AB})^2+(1+2|E_BE_C|+E_{BC})^2+(1+2|E_CE_A|+E_{CA})^2$$
$$\leq 6(1+|E_A|)\cdot(1+|E_B|)\cdot(1+|E_C|)$$

arXiv:1906.06495
Nature Commun. in press

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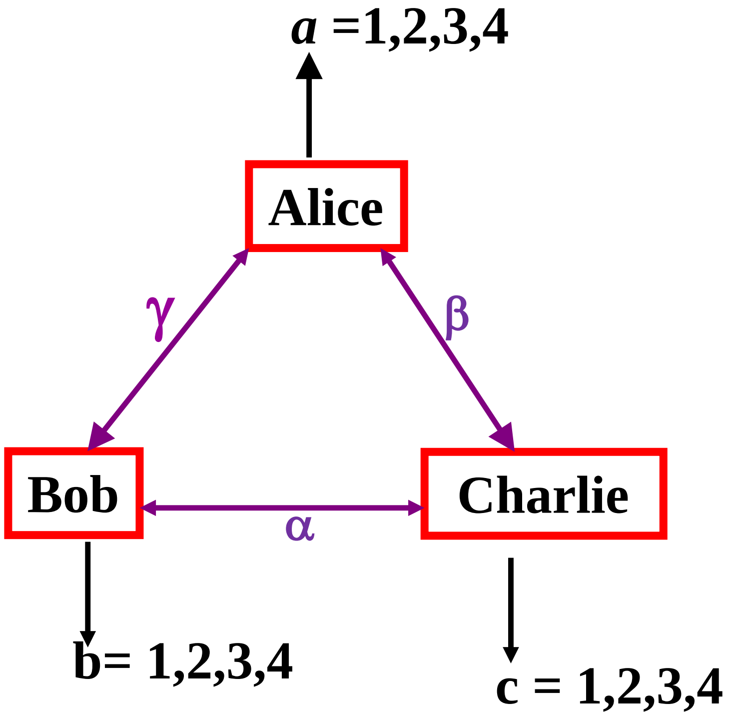
# 4-outcome Triangle




- α,β,γ = $\phi^+$ and Bell State Measurements ⇒ p(a,b,c) is local
- Are there other «natural» 2-qubit measurements ?

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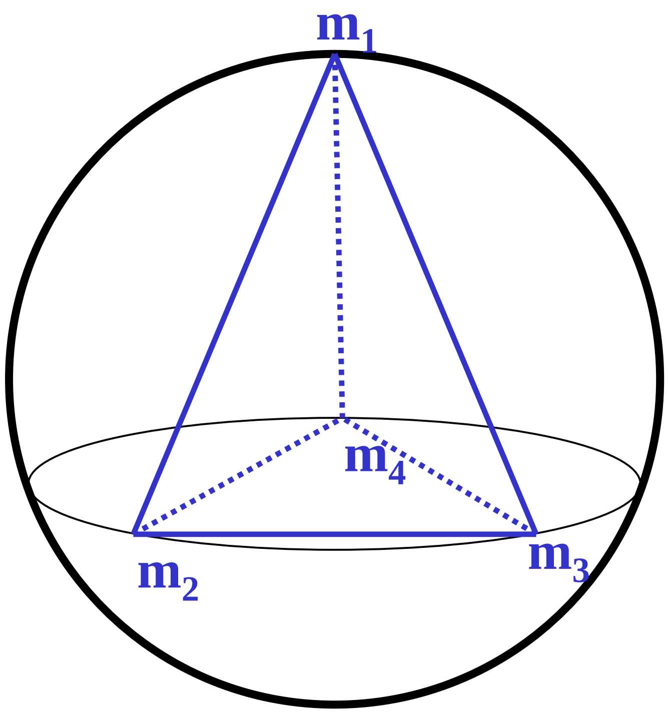
# The Elegant Joint Measurement (EJM)

GAP Optique **Geneva University**




**Look for 4 partially entangled and mutually orthogonal states with same degrees of entanglement and with partial states along the vertices of the tetrahedron.**

$$\left|\Phi_j\right\rangle = c_0\left|\vec{m}_j,-\vec{m}_j\right\rangle + q_0\left|-\vec{m}_j,\vec{m}_j\right\rangle$$

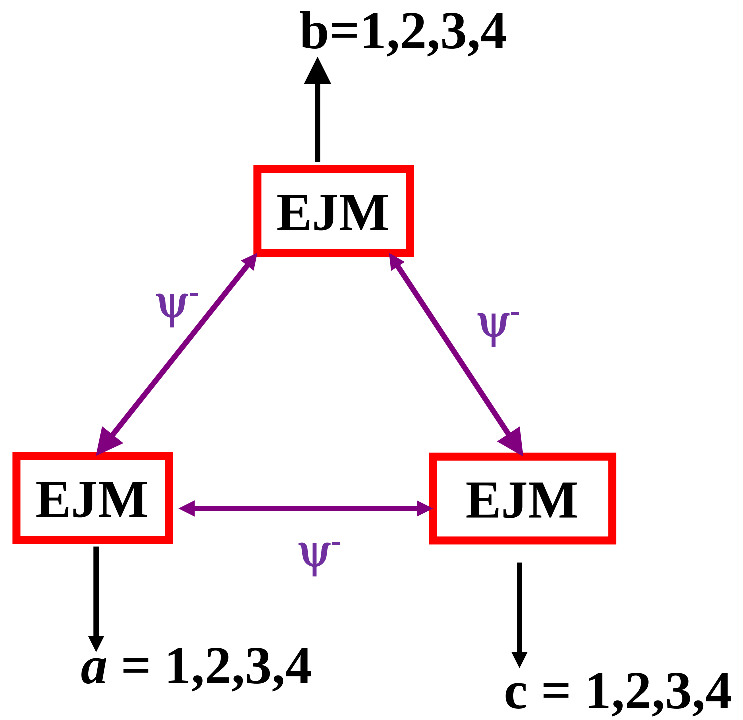
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}\left|\vec{m}_j,-\vec{m}_j\right\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}}\left|-\vec{m}_j,\vec{m}_j\right\rangle$$

$\left\langle\Phi_j\middle|\Phi_i\right\rangle = \delta_{ji}$ & $c_0, c_1$ are real

$$Tr_B\left(\left|\Phi_j\right\rangle\left\langle\Phi_j\right|\right) = \frac{1}{2}\left(1+\frac{\sqrt{3}}{2}\vec{m}_j\right) \qquad Tr_A\left(\left|\Phi_j\right\rangle\left\langle\Phi_j\right|\right) = \frac{1}{2}\left(1-\frac{\sqrt{3}}{2}\vec{m}_j\right)$$

NG, Entropy, «25 years of Q teleportation», 21,325 (2019)

# The Elegant Distribution




$$\Rightarrow \mathbf{p(a,b,c)} = \begin{cases} \frac{25}{256} & \text{if} \quad a = b = c \\ \frac{1}{256} & \text{if} \quad a = b \neq c, \quad a = c \\ \frac{5}{256} & \text{if} \quad a \neq b \neq c \neq a \end{cases}$$

**Challenge 4:**
**prove that the elegant distribution is local / non-local.**

NG, Entropy, «25 years of Q teleportation», 21,325 (2019)

# Distributions invariant under output permutations: π(0,1,2,3)

$$\mathbf{p(a,b,c)} = \begin{cases} p_1 & \text{if} \quad a=b=c \\ p_2 & \text{if} \quad a=b\neq c, \quad a=c\neq b, \quad a\neq b=c \\ p_3 & \text{if} \quad a\neq b\neq c\neq a \end{cases}$$

**Normalization ⇒ $4p_1+36p_2+24p_{3}=1$**

**Write the outcomes as 2 bits, e.g. $a=a_0+2a_1$ and look for the correlators:**

**Switching to ”physicists” bits ±1 all correlators vanish except**
**<a0b0> = <a1b1> = 4p1+4p2-8p3 = E2**
**<a0a1b0b1> = 4p1+4p2-8p3 = E2**
**<a0a1b0c1> = <a0a1b1c0> = 4p1-12p2+8p3 = E3**

**Only correlators with an even number of bits and an even number of indices 1 do not vanish.**

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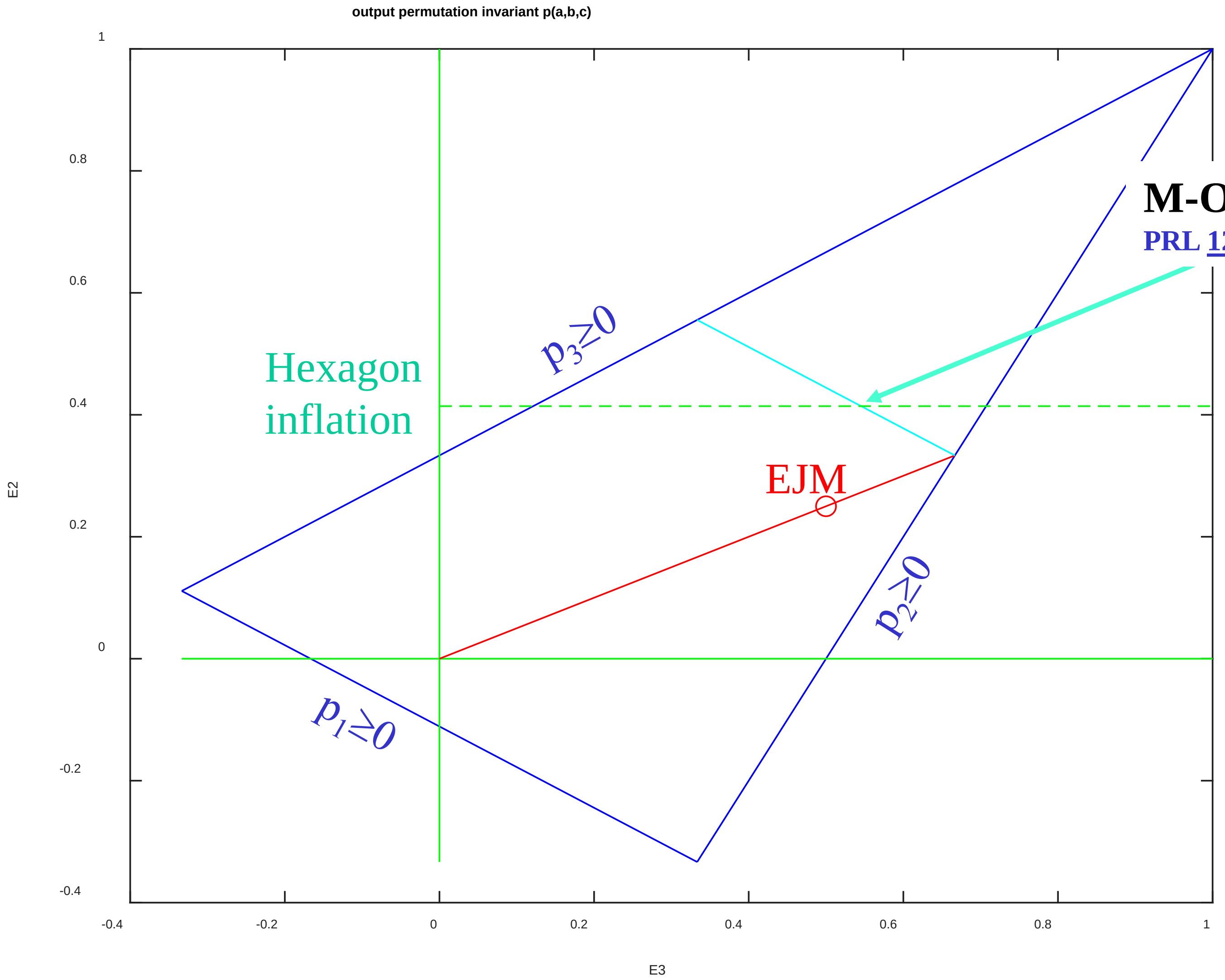
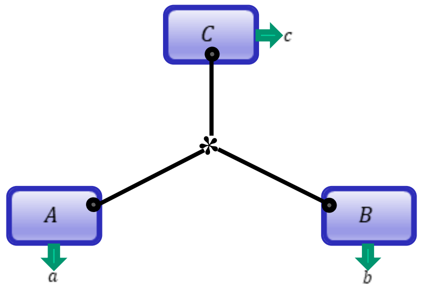
# Distributions invariant under output permutations: $\pi(0,1,2,3)$

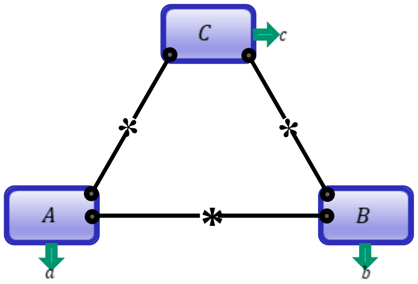



**M-O Renou et al.**
**PRL 123, 070403 (2019)**

**GAP Optique Geneva University**

# The Finner problem




## Finner problem

- $P_A(0) = P_B(0) = P_c(0)$ fixed
- Maximize $P(000)$?
- ➢ Network ≡ resource

## Finner Inequalities

- GHZ network

$$P(000) \leq \left(P_A(0)P_B(0)P_C(0)\right)^{1/3}$$

**Triangle network?**

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# The Finner problem

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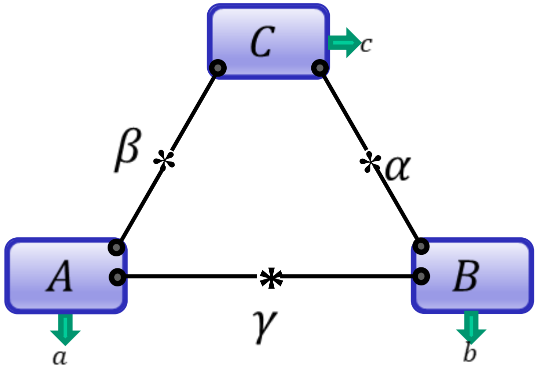
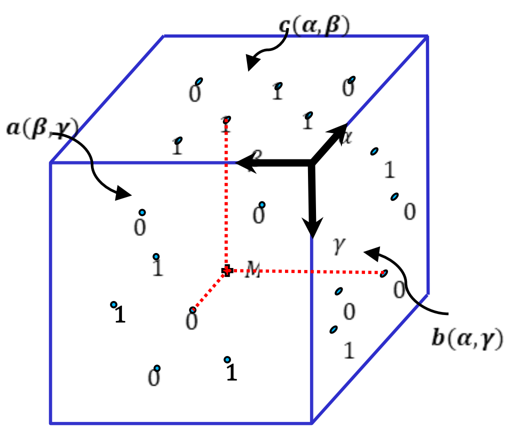



## Cube representation of all local strategies

- $\alpha, \beta, \gamma \in [0,1]$ uniformly random
- Deterministic outputs $a(\beta,\gamma), b(\gamma,\alpha), c(\alpha,\beta)$
- $p_{abc}$: Volume of $(a, b, c)$ in the cube
- Marginal $p_a^A$ : area of $a$ on $A$ face

## Geometrical Inequality

- Loomis Witney Inequality:

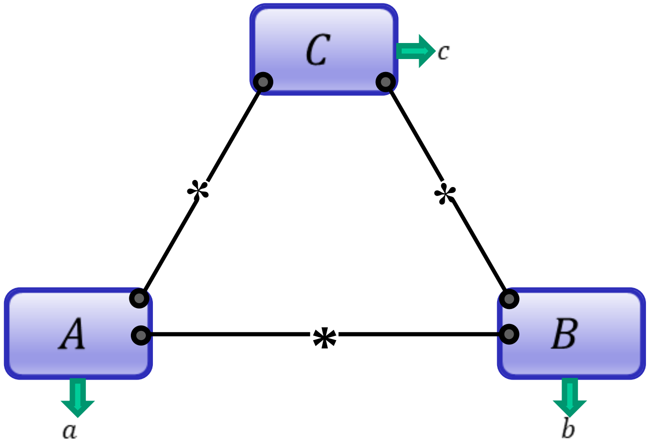
  $$\text{"Volume}^2 \leq \prod \perp\text{Areas"}$$

- Here, becomes Finner inequality:

  $$p_{abc}^2 \leq p_a^A p_b^B p_c^C$$

# The Finner problem






**The triangle Finner inequality**

$$P(000) \leq \left(P_A(0)P_B(0)P_C(0)\right)^{1/2}$$

**Validity**

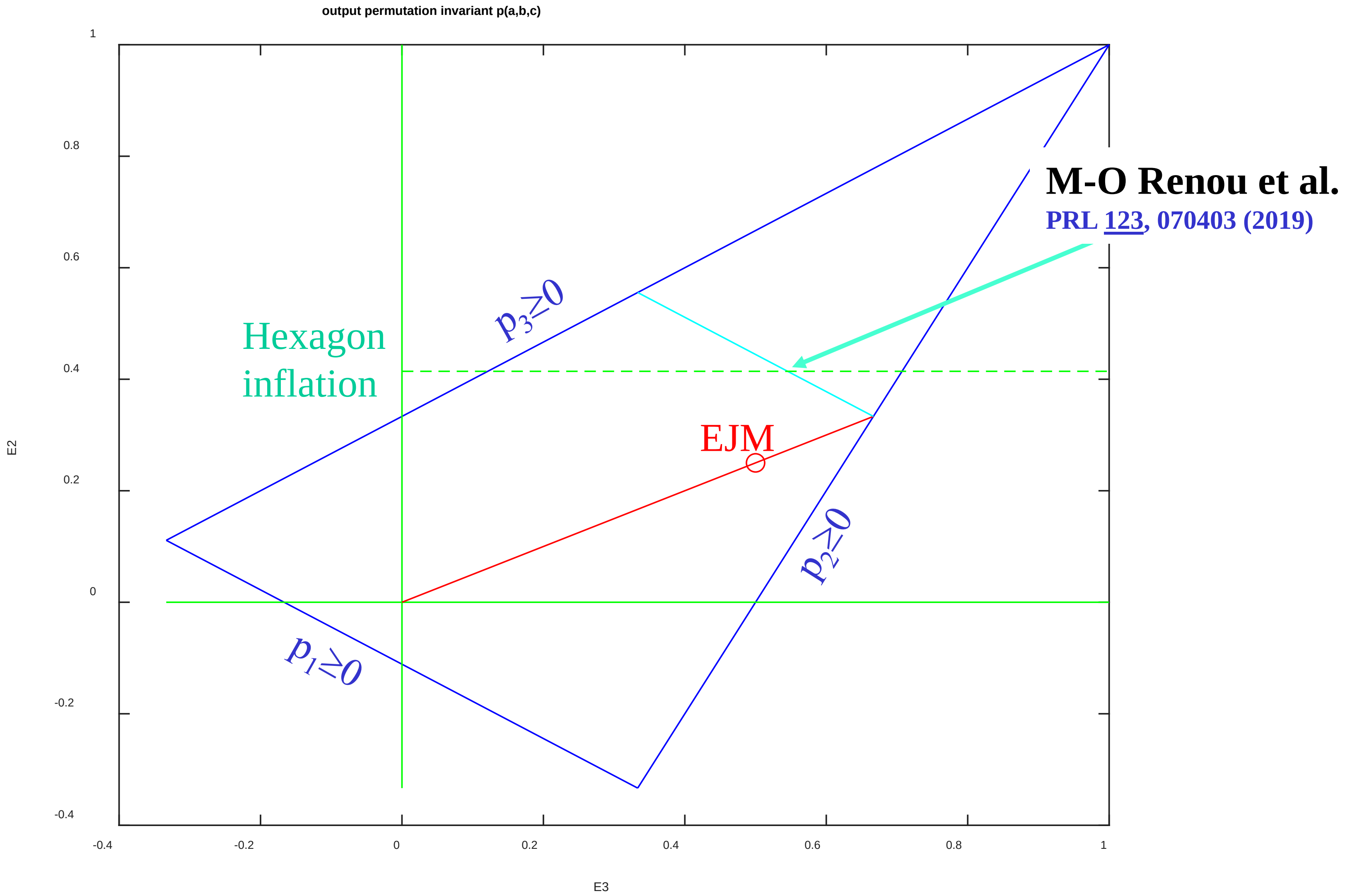
- Local resources

**Theorem 1:** Finner Inequality holds in $Q$

**Theorem 2:** The Finner inequality is valid for wirings of PR-Boxes

M.O. Renou, Y. Wang, S. Boreiri, S. Beigi, N. Gisin, and N. Brunner, PRL 123, 070403 (2019)

# Distributions invariant under output permutations: $\pi(0,1,2,3)$

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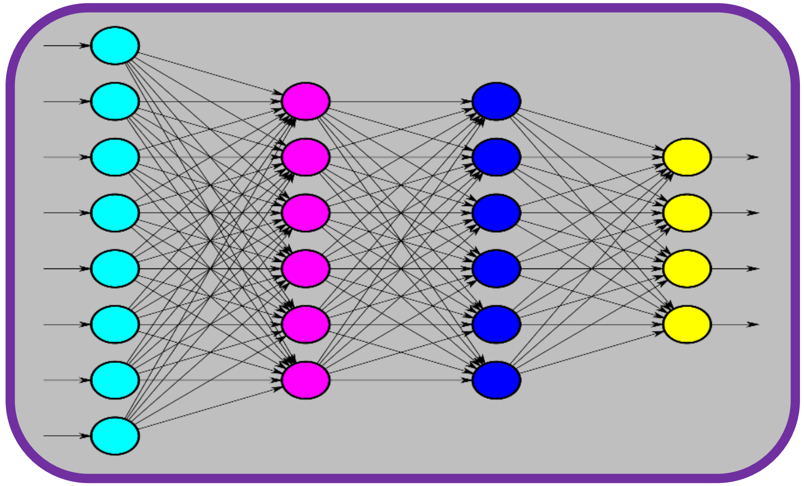
# Machine Learning

# Neural networks

Tamas Krivachy



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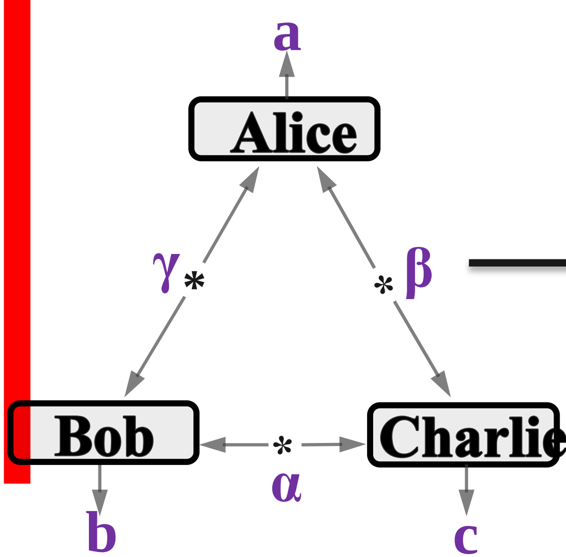
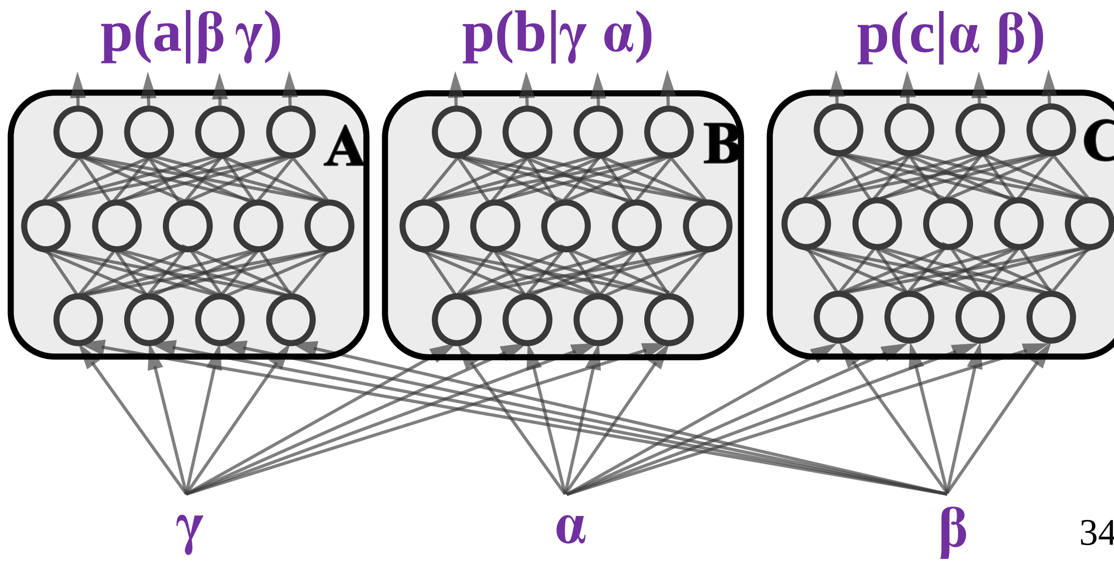
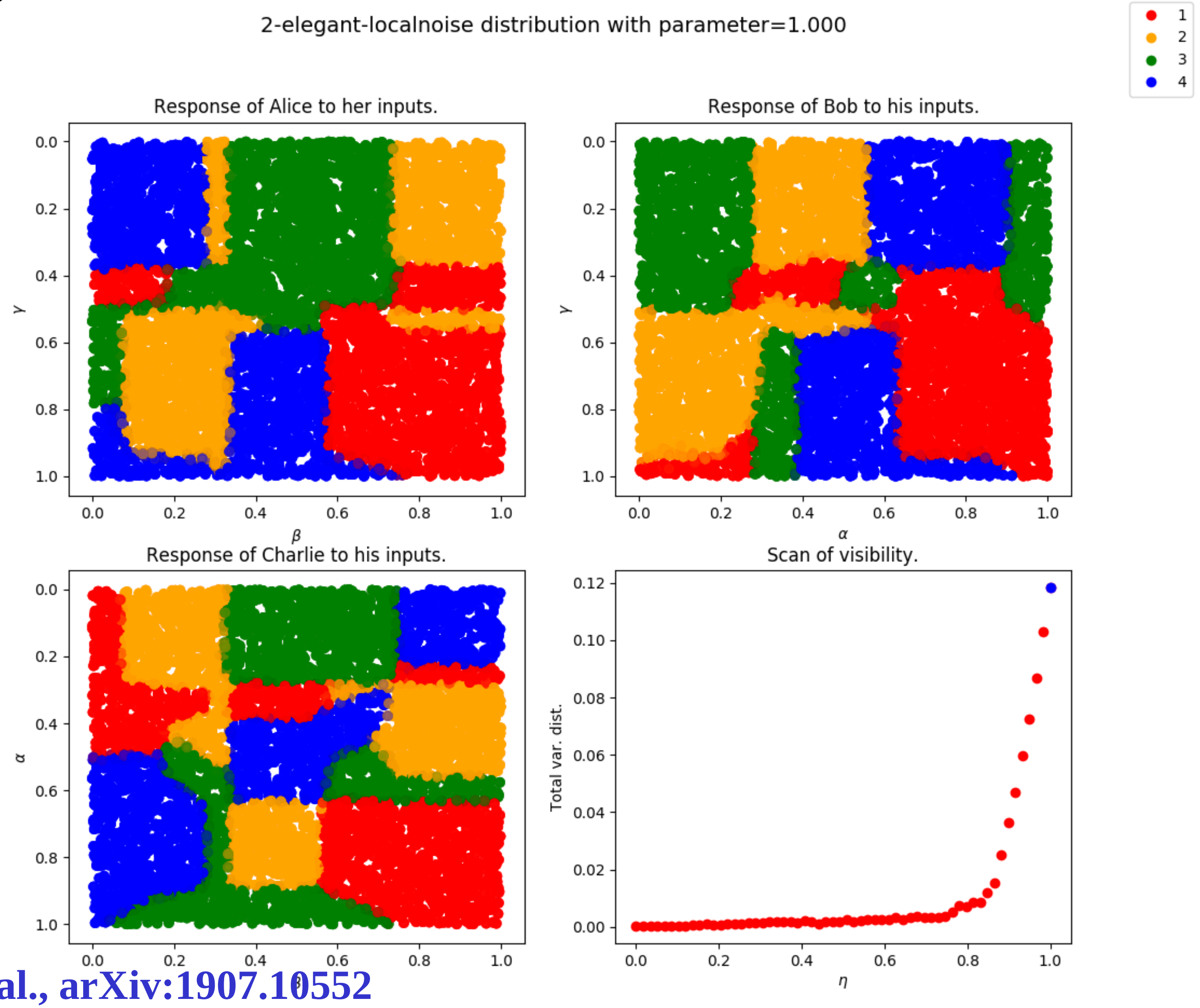


# Elegant distribution & Machine Learning

Tamas Krivachy

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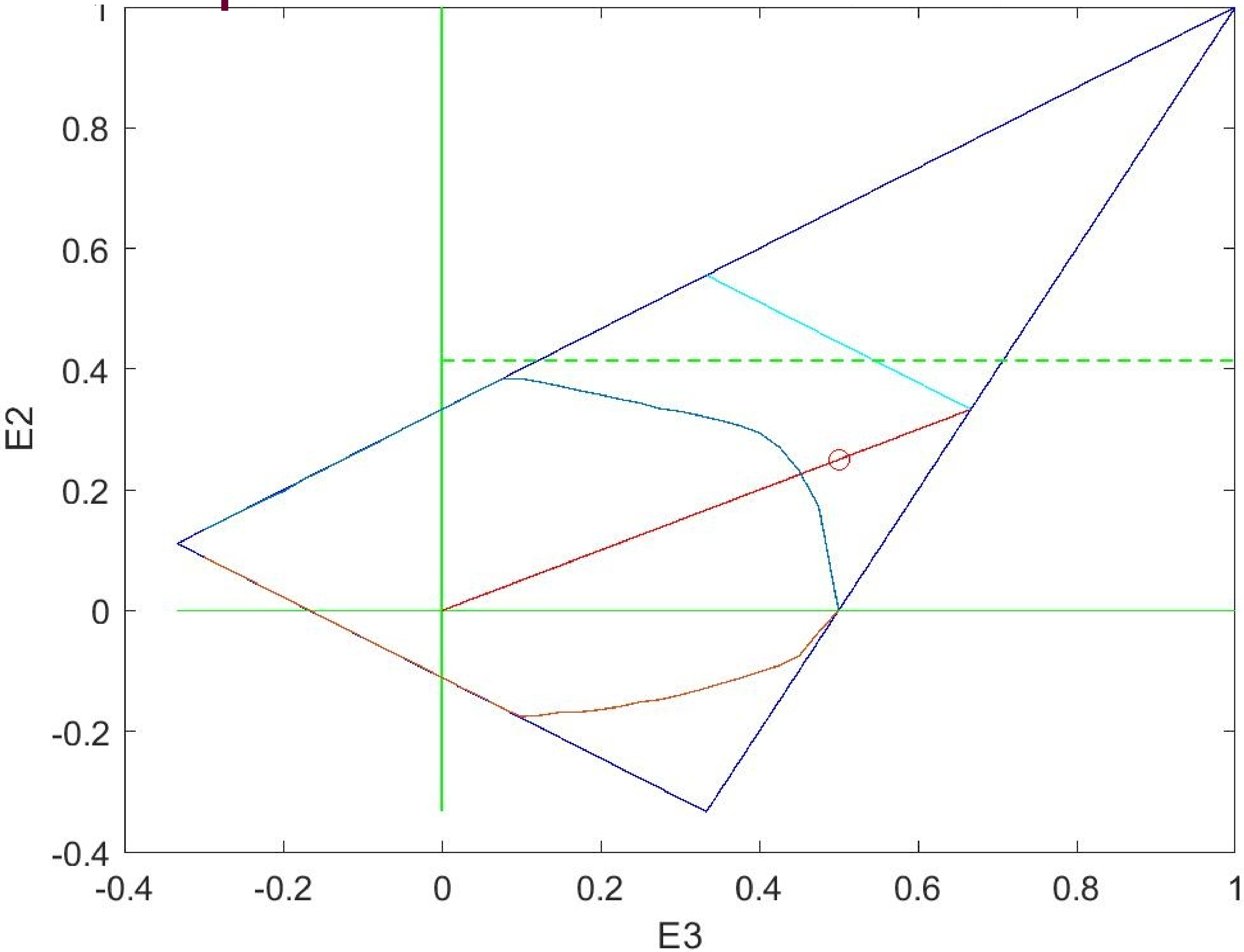
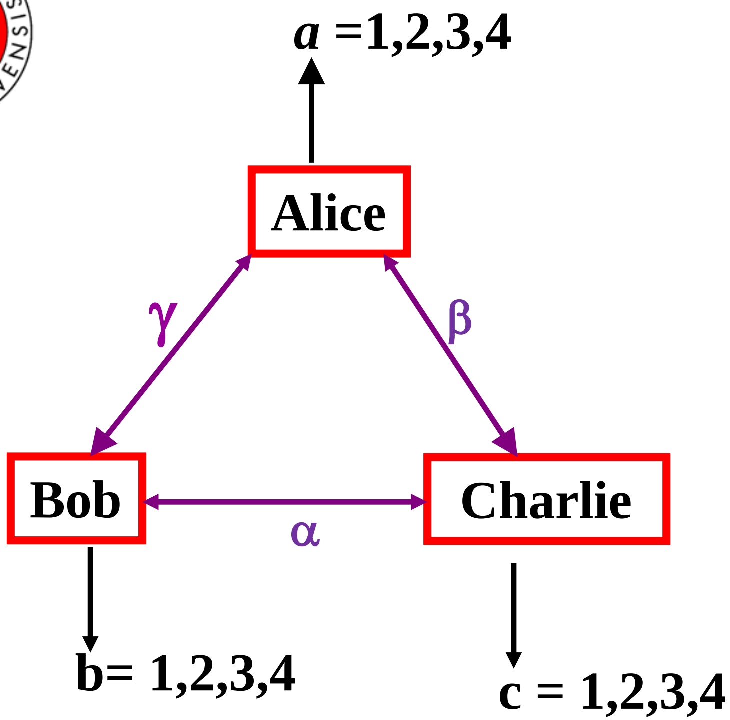
Kryvachy et al., arXiv:1907.10552

# Assume all local sources and response functions are identical




JD Bancal, N. Brunner, Cai Yu, NG, work in progress

# The Salman Triangle




$$\alpha,\beta,\gamma = \phi^{+}$$

**Eigenstates of measurements:**

$$|\varphi_1\rangle = |0,1\rangle \qquad |\varphi_2\rangle = |1,0\rangle$$

$$|\varphi_3\rangle = c_1|0,0\rangle + s_1|1,1\rangle$$

$$|\varphi_4\rangle = c_2|0,0\rangle + s_2|1,1\rangle$$

$$P\left(|0,1\rangle, |0,1\rangle, c\right) = 0$$

**<u>Theorem</u>: For $c_1$ large enough, the quantum distribution p(a,b,c) is non-local:**

$$p(a,b,c) \neq \sum_{\alpha\beta\gamma} p(\alpha)p(\beta)p(\gamma)p(a|\beta\gamma)p(b|\gamma\alpha)p(c|\alpha\beta)$$

Renou,Bäumer,Boreiri,Brunner,NG & Beigi, arXiv:1905.04902, PRL <u>123</u>, 140401 (2019)

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# The Salman Triangle



**Eigenstates of measurements:**

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**<u>Theorem</u>: For $c_1$ large enough, the quantum distribution p(a,b,c) is non-local.**

$$P\left(|0,1\rangle, |0,1\rangle, c\right) = 0$$

**<u>Lemma</u> 1: Denote X,Y,Z the 3 sets of local variables α,β,γ. These sets partition in two: $X=X_0\cup X_1$, $Y=Y_0\cup Y_1$, $Z=Z_0\cup Z_1$ s.t.**

$$a(\gamma,\beta) = |0,1\rangle \quad \Leftrightarrow \quad \beta \in Y_0 \;\&\; \gamma \in Z_1$$

$$a(\gamma,\beta) = |1,0\rangle \quad \Leftrightarrow \quad \beta \in Y_1 \;\&\; \gamma \in Z_0$$

**<u>Lemma</u> 2: For** $c_1 > \frac{1}{2}\sqrt{2k - 6/k} \approx 0.886, \quad where\ k = \sqrt[3]{9 + 6\sqrt{3}}$

**no 3-local model reproduces $p(a,b,c\in\{\varphi_3,\varphi_4\}^3)$**

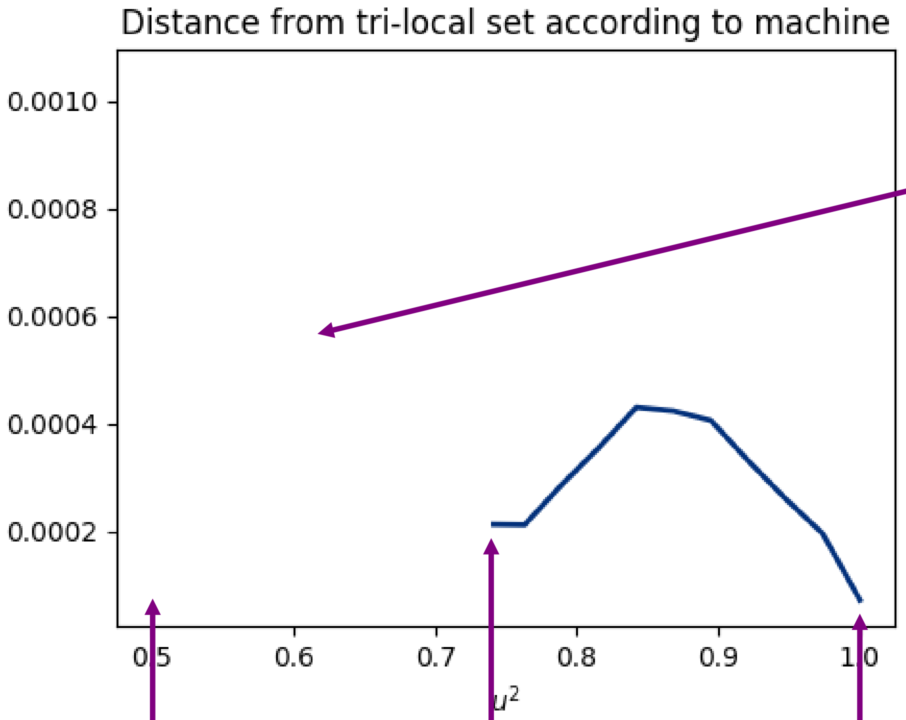
**<u>Challenge 5</u>:**

**How robust – noise resistant – is the Salman distribution?**

arXiv:1905.04902, PRL <u>123</u>, 140401 (2019)

# The Salman Triangle

## Machine learning




**There are local models with trits**

**Challenge 6: Prove Salman distribution non-local below the critical $c^2$**

**Challenge 7:**

**i) Is the Salman truly "genuinely triangle" ? Or can it be realized with only 1 or 2 entangled states ?**

**ii) Define "genuinely triangle": e.g. entangled states & measurements ?**

# Conclusions - Challenges

1. **Find a bi-locality scenario with critical visibility per singlet ≥ 1/√2.**
2. **What is in the top of the E2 vs E3 set of the binary triangle: is this local? Quantum?**
3. **Find a triangle inequality that involves 3-party correlators.**
4. **Prove that the elegant distribution is local / non-local.**
5. **How robust – noise resistant – is the Salman distribution?**
6. **Prove Salman's distribution is non-local below the critical c.**
7. **Define «genuinely triangle» and apply to Salman's distr.**


**arXiv:1901.08287 (Finner ineq.), PRL 123, 070403 (2019)**
**arXiv:1906.06495 (Salman Triangle), PRL 123,140401 (2019)**
**arXiv:1905.04902 (NSI principle), Nature Commun. In press**
**arXiv:1907.10552 (Machine learning)**


**Prove the existence of randomness without inputs**

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